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Gravitational waves in the dynamic medium of reference theory

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Abstract The theory of the dynamic medium of reference (DMR) has already been presented in several articles, in particular, Pignard [Phys. Essays **32**, 422 (2019]. The objective of this article is to present gravitational waves within the framework of the DMR theory. For this, an important relation is established between the gravitational potential and the speed of the flux of the medium. This relation makes it possible to deduce two differential equations verified by the speed of the flux of the medium, one relating to the stationary part and the other to the variational part, which corresponds to gravitational waves. Solving these two equations provides the speed of the total flux of the medium. An application of the found formulas is carried out to a binary system, and the link with the metric tensor and the fundamental quadratic form of general relativity is established. Finally, in the DMR theory, gravitational waves are waves that propagate through the medium at the speed of light, not ripples of space-time. © *2022 Physics Essays Publication*.
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Résumé: La théorie du Milieu Dynamique de Référence (DMR) a déjà été présentée dans plusieurs articles, en particulier "Dynamic Medium of Reference: A new theory of gravitation" [Pignard, Phys. Essays **32**, 422, 2019]. Le présent article a pour objectif de présenter les ondes gravitationnelles dans le cadre de la théorie DMR. Pour cela une relation importante est établie entre le potentiel gravitationnel et la vitesse du flux du milieu. Cette relation permet de déduire deux équations différentielles vérifiées par la vitesse du flux du milieu, l'une portant sur la partie stationnaire, l'autre sur la partie variationnelle qui correspond aux ondes gravitationnelles. La résolution des ces deux équations fournit la vitesse du flux du medium total. Une application des formules trouvées est réalisée à un système binaire et le lien avec le tenseur métrique et la forme quadratique fondamentale de la relativité générale est établie. Enfin, dans la théorie DMR, les ondes gravitationnelles sont des ondes qui se propagent dans le medium à la vitesse de la lumière et non pas des rides de l'espace-temps.

Key words: Dynamic Medium of Reference; Gravitation; Gravitational Waves; Gravitational Potential; Flux of the Medium.

I. SUCCINCT PRESENTATION OF THE THEORY OF THE DYNAMIC MEDIUM OF REFERENCE

Important preliminary remark:

This article does not present the theory of the dynamic medium of reference (DMR).

The theory of the DMR has already been presented in several articles,[1–6] in particular, "Dynamic Medium of Reference: A new theory of gravitation,"[1] which is strongly recommended to read in order to understand this article.

The theory of the DMR[1] introduces a **dynamic nonmaterial** medium, which is present in the whole Universe.

The characteristics of this medium are:

- This medium enables one to deduce a preferred frame of reference (PFR) or rather a reference in the whole universe and at all scales.
- This reference enables one to obtain a privileged time. The present moment is universal, that is to say the same in the whole universe.
- This medium is also the medium of propagation of light.

This medium verifies the principle of reciprocal action:

- The medium is distorted by matter and energy like the space-time of general relativity.
- The warping of this medium determines the trajectories of the particles (material particles and light particles).

The presence of a massive body creates a flux of the medium (centripetal that is to say directed toward the center of gravity of the massive body)

$$\text{of speed } V_{\text{flux}} = \sqrt{\frac{2GM}{r}}, \tag{1}$$

$$\text{and acceleration } \gamma_{\text{flux}} = \frac{GM}{r^2}, \tag{2}$$

where r refers to the distance to the center of gravity of the massive body.

In the framework of Lorentz/Poincaré theory, in the absence of a gravitational field, material clocks (in the reference frame R) undergo a physical dilatation of their period according to their speed with respect to the PFR according to the formula

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$$T = \gamma.T_0 \quad \text{with } \gamma = \left(1 - \frac{V^2_{R/\text{PFR}}}{c_0^2}\right)^{-1/2}. \tag{3}$$

Within the framework of Lorentz/Poincaré theory, in the absence of a gravitational field, material rulers (in the reference frame R) undergo a physical contraction of their length according to their speed with respect to the PFR according to the formula

$$L = L_0/\gamma \quad \text{with } \gamma = \left(1 - \frac{V^2_{R/\text{PFR}}}{c_0^2}\right)^{-1/2}. \tag{4}$$

In the presence of a massive body of mass M, the speed of the flux of the medium takes the following expression in the frame of reference linked to the massive body and at a distance r from the center of gravity of the massive body:[1]

$$\overrightarrow{V_{\text{flux}}} = -\sqrt{\frac{2GM}{r}}\overrightarrow{u_r}, \tag{5}$$

where $\overrightarrow{u_r}$ denotes the unit radial vector directed toward the exterior of the massive body.

The effects undergone by material clocks and rulers due to their speed with respect to the medium (which allows one to define the PFR) are the same as the effects they undergo by the centripetal movement of the medium due to a massive body. Since clocks and rulers are assumed to be fixed with respect to the massive body, the centripetal movement of the medium (of speed V_{flux}) with respect to the center of gravity of the massive body can be interpreted as a movement of the clocks and rulers with respect to the medium.

The equivalence between the movement of the clocks and rulers with respect to the medium and the movement of the medium with respect to the clocks and rulers is a new way of stating **the principle of equivalence**.

In the presence of a massive body of mass M, material clocks undergo a physical dilatation of their period according to the following formula:[1]

$$T = T_0.K(r) \quad \text{with} \quad K(r) = \left(1 - \frac{V^2_{\text{flux}}}{c_0^2}\right)^{-1/2} = \left(1 - \frac{2GM}{c_0^2.r}\right)^{-1/2}. \tag{6}$$

In the presence of a massive body of mass M, material rulers undergo a physical contraction of their length according to the following formula:[1]

$$L = \frac{L_0}{K(r)} \quad \text{with} \quad K(r) = \left(1 - \frac{V^2_{\text{flux}}}{c_0^2}\right)^{-1/2} = \left(1 - \frac{2GM}{c_0^2.r}\right)^{-1/2}. \tag{7}$$

Light is slowed down by a gravitational field, and the expression of its speed is[1]

$$c = \frac{c_0}{K\sqrt{1 + (K^2 - 1)\cos^2\beta}}. \tag{8}$$

We name $\beta = (\overrightarrow{u_r}, \overrightarrow{c})$ the angle between the unit radial vector $\overrightarrow{u_r}$ and the speed vector of light $\overrightarrow{c}$. The vector $\overrightarrow{c_0}$ represents the speed vector of light if there was not any massive body.

In the case of a radial trajectory of the light, we have the following simple expression:

$$c = \frac{c_0}{n(r)} = \frac{c_0}{K^2(r)}. \tag{9}$$

If we call ε_0 the permittivity and μ_0 the permeability of the medium without gravitational field, then we have $c_0 = (\varepsilon_0\mu_0)^{-1/2}$.

If we call ε the permittivity and μ the permeability of the medium in the presence of a gravitational field created by a massive body of mass M, then we have $c = (\varepsilon\mu)^{-1/2}$

$$\text{with} \quad \varepsilon = \varepsilon_0.\varepsilon_r = \varepsilon_0\left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}, \tag{10}$$

$$\text{and} \quad \mu = \mu_0.\mu_r = \mu_0\left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}. \tag{11}$$

The refractive index is given by the formula

$$n = \sqrt{\varepsilon_r\mu_r} = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}, \tag{12}$$

$$\text{with} \quad \varepsilon_r = \varepsilon/\varepsilon_0 = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}, \tag{13}$$

$$\text{and} \quad \mu_r = \mu/\mu_0 = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}. \tag{14}$$

All these formulas show that the medium is related to electricity, magnetism, electromagnetism, and gravitation.

(a) Case of the photon

The exploitation of the previous results makes it possible to establish the Lagrangian formulation of the trajectory of a photon[1]

$$L = K^2\left[c_0^2 - \left(K^2\frac{dr}{dt}\right)^2 - \left(Kr\frac{d\phi}{dt}\right)^2\right] = 0 \tag{15}$$

that we can write

$$\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\phi}{dt}\right)^2 = c_0^2. \tag{16}$$

After the development of calculations, we end up with the following equation:[1]

$$\frac{d^2u}{d\phi^2} + u = \frac{3GM}{c_0^2}u^2 \quad \text{with} \quad u = 1/r. \tag{17}$$

This equation, which is exactly that provided by general relativity,[7–11] makes it possible to determine the deflection of light rays by a massive body, for example, the Sun, and also by clusters of galaxies (gravitational lens, gravitational mirage, Einstein ring).

(b) Case of a material particle

The exploitation of the previous results makes it possible to establish the Lagrangian formulation of the trajectory of a material particle[1]

$$c_0^2 - L = K^2\left[\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\phi}{dt}\right)^2 - \frac{2GM}{r} - C^2\right] = 0 \tag{18}$$

that we can write

$$\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\varphi}{dt}\right)^2 = \frac{2GM}{r} + C^2 = V_{\text{flux}}^2 + C^2. \tag{19}$$

After the development of calculations, we end up with the following equation:[1]

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{A^2} + \frac{3GM}{c_0^2}u^2 \quad \text{with} \quad u = 1/r. \tag{20}$$

This equation, which is exactly that provided by general relativity,[7–11] makes it possible to determine the trajectory of the planets of the solar system and, in particular, the precession of the perihelion of Mercury.

II. RELATIONSHIP BETWEEN THE GRAVITATIONAL POTENTIAL AND THE SPEED OF THE FLUX OF THE MEDIUM

In the article, "Theory of the dynamic medium of reference: exterior case and interior case,"[4] we saw that the flux of the medium generated by a homogeneous body of mass M has for acceleration and speed, the following expressions in the exterior case and the interior case (we add on the last line the Newtonian gravitational potential) (Table I).

We then observed the following relation between the gravitational potential and the speed of the flux of the medium in the external case and the internal case:

$$V_{\text{flux}}^2(r) = -2\phi(r). \tag{21}$$

We will demonstrate this relationship.

First, let us express the relationship between the acceleration and the speed of the flux of the medium

$$\gamma_{\text{flux}} = \frac{dV_{\text{flux}}}{dt} = \frac{dV_{\text{flux}}}{dr}\frac{dr}{dt} = \frac{dV_{\text{flux}}}{dr}V_{\text{flux}} = \frac{d}{dr}\left(\frac{V_{\text{flux}}^2}{2}\right). \tag{22}$$

The speed of the flux of the medium is, therefore, given by the following formula:

$$V_{\text{flux}}^2 = \int 2\gamma_{\text{flux}}dr. \tag{23}$$

Gravitational acceleration is related to gravitational potential by the relationship

$$\overrightarrow{\gamma_G} = -\overrightarrow{\nabla}\phi = -\frac{d\phi}{dr}\overrightarrow{u_r}. \tag{24}$$

Knowing that the gravitational acceleration is physically the acceleration of the flux of the medium, Eqs. (22) and (24) allow us to find Eq. (21) in the form

$$\frac{V_{\text{flux}}^2}{2} = -\phi. \tag{25}$$

The Poisson equation is written as follows:

$$\Delta\phi = 4\pi G\rho, \tag{26}$$

where $\Delta\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$.

Thus, we obtain the following fundamental relation on the speed of the flux of the medium:

$$\Delta\left(V_{\text{flux}}^2\right) = -8\pi G\rho. \tag{27}$$

III. STATIONARY PART OF THE GRAVITATIONAL FIELD

The stationary part of the gravitational field, characterized by the speed of the flux of the medium V_{flux}, satisfies the following differential equation:

$$\triangle\left(V_{\text{flux}}^2\right) = -\frac{8\pi G}{c^2}\left(\rho c^2\right). \tag{28}$$

The stationary solution of this differential equation, far from the source, is

TABLE I. Acceleration and speed of the flux of the medium in the exterior case and the interior case.

	Exterior case	Interior case
Acceleration of the flux of the medium	$\gamma_{\text{flux}} = \frac{GM}{r^2}$	$\gamma_{\text{flux}} = \frac{GM}{R^3}r$
Speed of the flux of the medium	$V_{\text{flux}} = \sqrt{\frac{2GM}{r}}$	$V_{\text{flux}} = \sqrt{\frac{GM}{R}\left(3 - \frac{r^2}{R^2}\right)}$
Newtonian gravitational potential	$\phi(r) = -\frac{GM}{r}$	$\phi(r) = -\frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right)$

$$V^2_{\text{flux}(0)} = \frac{2G}{r}\int \rho dV = \frac{2GM}{r} \quad \text{by setting} \quad M = \int \rho dV. \tag{29}$$

IV. VARIATIONAL PART OF THE SPEED OF THE FLUX OF THE MEDIUM CORRESPONDING TO GRAVITATIONAL WAVES

The variational part of the speed of the flux of the medium V_{flux}, corresponding to gravitational waves, verifies the following differential equation:

$$\Box\left(V^2_{\text{flux}(i,j)}\right) = -\frac{8\pi G}{c^2}\rho[v_i v_j - < v_i v_j >], \tag{30}$$

with $\Box A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} - \frac{\partial^2 A}{c^2 \partial t^2} = \Delta A - \frac{\partial^2 A}{c^2 \partial t^2}$.

The tensor $[v_i v_j - < v_i v_j >] =$

$$\begin{bmatrix} v_x^2 - < v_x^2 > & v_y v_x - < v_y v_x > & v_z v_x - < v_z v_x > \\ v_x v_y - < v_x v_y > & v_y^2 - < v_y^2 > & v_z v_y - < v_z v_y > \\ v_x v_z - < v_x v_z > & v_y v_z - < v_y v_z > & v_z^2 - < v_z^2 > \end{bmatrix}$$

corresponds to the "variational" part of the speed tensor of the source, $<v_i v_j >$ designating the mean value of $v_i v_j$.

The usual method of obtaining the general solution of Eq. (30) is based on the use of Green's functions, similar to what is usually done in electromagnetism.

The solution of the differential equation (30) in the case of the compact source is written as

$$\left[V^2_{\text{flux}(i,j)}\right] = \frac{2G}{c^2 r}\int [v_i v_j - < v_i v_j >]\rho dV. \tag{31}$$

In the approximation of the far-from-source solution, we have

$$\left[V^2_{\text{flux}(i,j)}\right] = \frac{2GM}{c^2 r}[v_i v_j - <v_i v_j>] \quad \text{by setting} \quad M = \int \rho dV. \tag{32}$$

The expressions of Eqs. (31) and (32) correspond only to the variational part of the flux of the medium.

Only the **speed of the total flux of the medium** corresponds to the real, physical flux of the medium and is given by the following formula where Id represents the identity matrix:

$$\left[V^2_{\text{flux tot}(i,j)}\right] = V^2_{\text{flux}(0)} Id + \left[V^2_{\text{flux}(i,j)}\right], \tag{33}$$

$$\left[V^2_{\text{flux tot}(i,j)}\right] = \frac{2GM}{r} Id + \frac{2GM}{c^2 r}[v_i v_j - < v_i v_j >]. \tag{34}$$

For the propagation of gravitational waves, which are a disturbance of the medium propagating at the speed of light, it is necessary to use the delayed time $t_r = t - r/c$.

V. EXAMPLE OF A BINARY SYSTEM

We consider a source made up of two massive bodies S_1 and S_2 each of mass M in nonrelativistic motion of rotation at angular speed Ω on circular orbits of radius a around their common center of mass.

It is a simplified model of a binary system in which the gravitational interaction keeps massive bodies in orbit.

By describing the motion in the Newtonian limit, we have[7] $\Omega = \sqrt{GM/(4a^3)}$.

The movements of the massive bodies of the source are described by the following vectors:

$$\overrightarrow{OS_1} = \begin{bmatrix} a.\cos(\Omega t) \\ a.\sin(\Omega t) \\ 0 \end{bmatrix} \quad \overrightarrow{V_{S_1}} = \begin{bmatrix} -a\Omega.\sin(\Omega t) \\ a\Omega.\cos(\Omega t) \\ 0 \end{bmatrix}$$

$$\overrightarrow{OS_2} = \begin{bmatrix} -a.\cos(\Omega t) \\ -a.\sin(\Omega t) \\ 0 \end{bmatrix} \quad \overrightarrow{V_{S_2}} = \begin{bmatrix} a\Omega.\sin(\Omega t) \\ -a\Omega.\cos(\Omega t) \\ 0 \end{bmatrix}.$$

We then have the following expressions that are the same for the first and the second body:

$v_x^2 = a^2\Omega^2 \sin^2(\Omega t) = -\frac{a^2\Omega^2}{2}\cos(2\Omega t) + \frac{a^2\Omega^2}{2}$ and $<v_x^2 >= \frac{a^2\Omega^2}{2}$,
$v_y^2 = a^2\Omega^2 \cos^2(\Omega t) = +\frac{a^2\Omega^2}{2}\cos(2\Omega t) + \frac{a^2\Omega^2}{2}$ and $<v_y^2 >= \frac{a^2\Omega^2}{2}$,
$v_x v_y = -a^2\Omega^2 \cos(\Omega t).\sin(\Omega t) = \frac{a^2\Omega^2}{2}\sin(2\Omega t)$ and $<v_x v_y >= 0$.

All the other terms of the speed tensor of the two bodies of the source are zero because $v_z = 0$.

The variational part of the speed of the flux of the medium V_{flux} corresponding to the gravitational waves generated by one of the two bodies of the source is, therefore, written

$$\left[V^2_{\text{flux}(i,j)}\right] = \frac{GM}{c^2 r}(a\Omega)^2 \begin{bmatrix} -\cos(2\Omega t_r) & \sin(2\Omega t_r) & 0 \\ \sin(2\Omega t_r) & \cos(2\Omega t_r) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{35}$$

where $t_r = t - r/c$ is the delayed time.

The speed of the total flux of the medium generated by the binary system is then given by the following formula:

$$\left[V^2_{\text{flux tot}(i,j)}\right] = 2V^2_{\text{flux}(0)} Id + 2\left[V^2_{\text{flux}(i,j)}\right], \tag{36}$$

$$\left[V^2_{\text{flux tot}(i,j)}\right] = \frac{4GM}{r}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{2GM}{r}\left(\frac{a\Omega}{c}\right)^2 \begin{bmatrix} -\cos(2\Omega t_r) & \sin(2\Omega t_r) & 0 \\ \sin(2\Omega t_r) & \cos(2\Omega t_r) & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{37}$$

VI. LINK BETWEEN GENERAL RELATIVITY AND THE THEORY OF DMR IN THE CASE OF THE BINARY SYSTEM

The fundamental quadratic form of general relativity is given by the following formula:

$$ds^2 = \sum_{i,j=0}^{3} g_{ij}dx^i dx^j = g_{00}(c.dt)^2 + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2 + 2g_{12}dx \cdot dy. \quad (38)$$

And the other terms being zero in the case of the binary system of the previous part.

The link between the fundamental quadratic form of general relativity and the theory of DMR is given by the following expressions:[4]

$$g_{00} = 1 - \frac{2V^2_{\text{flux}(0)}}{c^2} = 1 - \frac{4GM}{c^2 r}, \quad (39)$$

$$g_{11} = -\left(1 - \frac{V^2_{\text{flux tot}(1,1)}}{c^2}\right)^{-1} \approx -1 - \frac{V^2_{\text{flux tot}(1,1)}}{c^2} \approx -1 - \frac{4GM}{c^2 r}\left(1 - \frac{1}{2}\left(\frac{a\Omega}{c}\right)^2 \cos(2\Omega t_r)\right), \quad (40)$$

$$g_{22} = -\left(1 - \frac{V^2_{\text{flux tot}(2,2)}}{c^2}\right)^{-1} \approx -1 - \frac{V^2_{\text{flux tot}(2,2)}}{c^2} \approx -1 - \frac{4GM}{c^2 r}\left(1 + \frac{1}{2}\left(\frac{a\Omega}{c}\right)^2 \cos(2\Omega t_r)\right), \quad (41)$$

$$g_{33} = -\left(1 - \frac{V^2_{\text{flux tot}(3,3)}}{c^2}\right)^{-1} \approx -1 - \frac{V^2_{\text{flux tot}(3,3)}}{c^2} \approx -1 - \frac{4GM}{c^2 r}, \quad (42)$$

$$g_{12} = g_{21} = -\frac{V^2_{\text{flux tot}(1,2)}}{c^2} = -\frac{2GM}{c^2 r}\left(\frac{a\Omega}{c}\right)^2 \sin(2\Omega t_r), \quad (43)$$

where $t_r = t - r/c$ is the delayed time.

We find expressions of g_{ij} equivalent to those of general relativity in the case of a binary system.

Indeed, the linearized metric tensor is written[7–9]

$$g_{ij} \approx \eta_{ij} + h_{ij}, \quad (44)$$

with $\eta_{00} = 1$ $\eta_{11} = -1$ $\eta_{22} = -1$ $\eta_{33} = -1$ the others η_{ij} being zero

$$\text{and } h_{00} = \frac{4\phi_0}{c^2} = -\frac{2V^2_{\text{flux}(0)}}{c^2} \quad h_{0i} = h_{i0} = 0 \quad \text{for i} \neq 0, \quad (45)$$

$$\text{and } h_{ij} = \frac{2\phi_{ij}}{c^2} = -\frac{V^2_{\text{flux tot}(i,j)}}{c^2} \quad \text{for} \quad i,j \geq 1 \quad (46)$$

which gives us

$$g_{00} \approx \eta_{00} + h_{00} \approx 1 - \frac{2V^2_{\text{flux}(0)}}{c^2}, \quad (47)$$

$$g_{11} \approx \eta_{11} + h_{11} \approx -1 - \frac{V^2_{\text{flux tot}(1,1)}}{c^2}, \quad (48)$$

$$g_{22} \approx \eta_{22} + h_{22} \approx -1 - \frac{V^2_{\text{flux tot}(2,2)}}{c^2}, \quad (49)$$

$$g_{33} \approx \eta_{33} + h_{33} \approx -1 - \frac{V^2_{\text{flux tot}(3,3)}}{c^2}, \quad (50)$$

$$g_{12} \approx \eta_{12} + h_{12} \approx -\frac{V^2_{\text{flux tot}(1,2)}}{c^2}, \quad (51)$$

$$g_{21} \approx \eta_{21} + h_{21} \approx -\frac{V^2_{\text{flux tot}(2,1)}}{c^2}. \quad (52)$$

It is possible to write the fundamental quadratic form as follows:

$$ds^2 = \left(1 - \frac{4GM}{c^2 r}\right)(c.dt)^2 - \left(1 + \frac{4GM}{c^2 r}\right)dr^2 + \frac{2GM}{c^2 r}\left(\frac{a\Omega}{c}\right)^2 \left[\cos(2\Omega t_r)dx^2 - \cos(2\Omega t_r)dy^2 - 2\sin(2\Omega t_r)dx.dy\right], \quad (53)$$

where $t_r = t - r/c$ is the delayed time.

VII. CONCLUSION

The theory of the DMR makes it possible to find expressions similar to those of general relativity corresponding to gravitational waves.

However, in the context of general relativity, gravitational waves are ripples of space-time propagating at the speed of light.

In the frame of the DMR theory, gravitational waves are wavelets propagating in the medium at the speed of light, like wavelets propagate on a lake or waves propagate in the ocean.

Another important point is that the gravitational potential is nothing else than the squared speed of the flux of the medium up to a factor of -2, i.e., $V^2_{\text{flux}} = -2\phi$.

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