Black holes in the theory of the dynamic medium of reference

Olivier Pignard[a)]
16 Boulevard du Docteur Cathelin, 91160 Longjumeau, France

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Abstract: The aim of this article is to apply the theory of the dynamic medium of reference [O. Pignard, Phys. Essays **32**, 422 (2019)] to black holes and to find all the results of general relativity concerning black holes without rotation and without load. Among the most important results to which this article leads, we can mention: (1) The speed of the flux of the medium is greater than the speed of light inside the horizon of a black hole or even much greater than the speed of light at a distance from the center of the black hole much less than the radius of Schwarzschild. (2) In the hybrid coordinate system ($dr_{\text{Schwarzschild}}$, $dt_{\text{free fall}}$), the speed of light is established simply in relation to its propagation medium. (3) A photon emitted at an infinite distance from the black hole with speed c_0 arrives near the horizon of the black hole with a real speed zero. And yet the local measurement of the speed of the photon carried out with a material clock and a material ruler remains c_0. (4) Study of the possible orbits of a material particle around a black hole and the possibility of orbits of a photon around a black hole. © *2020 Physics Essays Publication.* [http://dx.doi.org/10.4006/0836-1398-33.4.395]

Résumé: Cet article a pour objectif d'appliquer la théorie du DMR (Dynamic Medium of Reference) [O. Pignard, Phys. Essays **32**, 422 (2019)] aux trous noirs et de retrouver tous les résultats de la relativité générale concernant les trous noirs sans rotation et sans charge. Parmi les résultats les plus importants auxquels aboutit cet article on peut mentionner: (1) La vitesse du flux du milieu est supérieure à la vitesse de la lumière à l'intérieur de l'horizon d'un trou noir voire bien supérieure à la vitesse de la lumière à une distance du centre du trou noir bien inférieure au rayon de Schwarzschild. (2) Dans le système de coordonnées hybrides ($dr_{\text{Schwarzschild}}$, $dt_{\text{free fall}}$) la vitesse de la lumière est établie simplement par rapport à son milieu de propagation. (3) Un photon émis à une distance infinie du trou noir avec la vitesse c_0 arrive au voisinage de l'horizon du trou noir avec une vitesse réelle nulle. Et pourtant la mesure locale de la vitesse du photon réalisée avec une horloge matérielle et une règle matérielle demeure c_0. (4) Etude des orbites possibles d'une particule matérielle autour d'un trou noir et de la possibilité d'orbites d'un photon autour d'un trou noir.

Key words: Dynamic Medium of Reference; Black Holes; General Relativity; Speed of Light; Speed of the flux of the medium; Photon.

I. PRESENTATION OF THE DYNAMIC MEDIUM OF REFERENCE

The theory of the dynamic medium of reference (DMR) introduces a **dynamic nonmaterial** medium that is present in the whole Universe.

The characteristics of this medium are:

- This medium enables one to deduce a preferred frame of reference or rather a REFERENCE in the whole Universe and at all scales.
- This REFERENCE enables one to obtain a privileged time. The present moment is universal, that is to say the same in the whole Universe.
- This medium is also the medium of propagation of light.
- This medium verifies the principle of reciprocal action:
  - The medium is distorted by matter and energy like the space-time of general relativity.
  - The warping of this medium determines the trajectories of the particles (material particles and light particles).

The presence of a massive body creates a flux of the medium (centripetal, that is to say, directed toward the center of gravity of the massive body) of **speed** $V_{\text{flux}} = \sqrt{2GM/r}$ and **acceleration** $\gamma_{\text{flux}} = GM/r^2$, where r refers to the distance to the center of gravity of the massive body.

II. SPEED OF THE FLUX OF THE MEDIUM INSIDE (OF SCHWARZSCHILD RADIUS) OF A BLACK HOLE

In general relativity, a stationary black hole, without rotation and without load, is described by the metric of Schwarzschild:[2]

$$ds^2 = \left(1 - \frac{2GM}{c_0^2 r}\right) c_0^2 dt^2 - \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta \cdot d\phi^2. \quad (1)$$

A black hole is bounded by the Schwarzschild radius $r_S = 2GM/c_0^2$ called the horizon of the black hole. Inside the horizon, nothing can escape from the black hole, not even light.

In the theory of the DMR,[1] a black hole generates a centripetal flux of the medium (radial and directed toward the

a)olivier_pacific@hotmail.fr

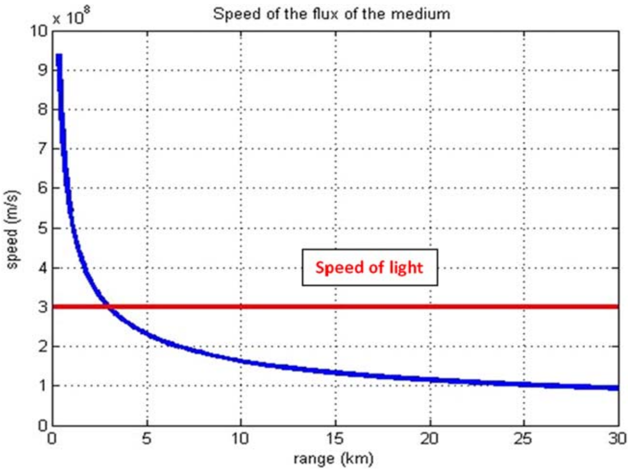

FIG. 1. (Color online) Speed of the flux of the medium.

center of the black hole) whose speed has for expression (Fig. 1)

$$V_{flux} = \sqrt{\frac{2GM}{r}}. \tag{2}$$

Beyond the horizon of a black hole ($r > r_s$) given by the Schwarzschild radius $r_S = 2GM/c^2$, the speed of the flux of the medium is lower than the speed of light.

However, inside the horizon of a black hole ($r < r_s$), the speed of the flux of the medium is greater than the speed of light.

Moreover, for a distance to the center of the black hole much less than the Schwarzschild radius ($r \ll r_s$), the speed of the flux of the medium is much greater than the speed of light ($V_{\rm flux} \gg c$).

For a distance r corresponding to the Planck length $L_P = \sqrt{Gh/(2\pi \cdot c^3)} = 1.616 \times 10^{-35}$m, the speed of the flux of the medium is

$$V_{\rm flux} = \sqrt{\frac{2GM}{L_P}} = 2.87 \times 10^{12}\sqrt{M}.$$

For a black hole of a solar mass ($\mathrm{Ms} = 2 \times 10^{30}$ kg), the speed of the flux of the medium is $V_{\rm flux} = 4 \times 10^{27}$ m/s.

Inside the horizon of a black hole is the only place in the current Universe where the speed of the flux of the medium is greater than the speed of light.

III. DEFINITION OF THREE FRAMES OF REFERENCE

Schwarzschild frame of reference[3]

This frame of reference is characterized by the coordinate system (t, r, ϑ, ϕ) involved in Eq. (1) and which allows a global description.

These coordinates are not directly measured, but they allow an exhaustive mapping of space-time established in a simple and unique system.

Sphere frame of reference[3]

This frame of reference is linked to the sphere with radius r and with center being the one of the considered black hole.

In this local "sphere" frame of reference, we have

$$dt_{\rm sphere} = \sqrt{1 - \frac{2GM}{c_0^2 r}}dt\,, \tag{3}$$

and

$$dr_{\rm sphere} = \frac{dr}{\sqrt{1 - \frac{2GM}{c_0^2 r}}}. \tag{4}$$

Free fall frame of reference[3]

This frame of reference is that of special relativity, where everything happens locally as if there was no gravitation. In this frame of reference, the variables are affixed with the index ff (free fall).

This free fall frame of reference is more general than the sphere frame of reference in at least two meanings:

- Lorentzian physics can be made valid there "longer" (by reducing the spatial extension, while the gravitational effect cannot be reduced on the sphere).
- It remains valid inside the black hole, for $r < r_s$, even though the concept of stationary sphere no longer exists.

IV. RADIAL TRAJECTORIES OF PHOTONS INSIDE A BLACK HOLE

To go from sphere coordinates to those of a free fall observer, one must perform a Lorentz transformation

$$dt_{\rm ff} = \gamma \cdot dt_{\rm sphere} - \gamma \frac{V_{\rm rel}}{c_0^2} dr_{\rm sphere}. \tag{5}$$

$V_{\rm rel}$ is the local relative speed between the two frames of reference. In the theory of DMR, it is simply the speed of an observer in free fall (compared with the reference frame of the black hole) and it is also the speed of the flux of the medium: $V_{\rm rel} = -V_{\rm flux} = -\sqrt{2GM/r}$.

The coefficient γ is therefore written: $\gamma = (1 - V_{\rm rel}^2/c_0^2)^{-1/2} = (1 - V_{\rm flux}^2/c_0^2)^{-1/2}$.

We also have

$$dt_{\rm sphere} = \sqrt{1 - \frac{2GM}{c_0^2 r}}dt = \sqrt{1 - \frac{V_{\rm flux}^2}{c_0^2}}dt \quad \text{and}$$

$$dr_{\rm sphere} = \frac{dr}{\sqrt{1 - \frac{2GM}{c_0^2 r}}} = \frac{dr}{\sqrt{1 - \frac{V_{\rm flux}^2}{c_0^2}}}.$$

So, we can write $dt_{\rm ff}$ as follows:

$$\begin{aligned} dt_{\rm ff} &= \gamma \sqrt{1 - \frac{V_{\rm flux}^2}{c_0^2}}dt + \gamma \frac{V_{\rm flux}}{c_0^2} \frac{dr}{\sqrt{1 - \frac{V_{\rm flux}^2}{c_0^2}}} \\ &= \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{-1/2} \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{1/2} dt \\ &\quad + \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{-1/2} \frac{V_{\rm flux}}{c_0^2} \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{-1/2} dr. \end{aligned}$$

Finally, we have

$$dt_{\rm ff} = dt + \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{-1} \frac{V_{\rm flux}}{c_0^2} dr. \quad (6)$$

In the article, "Dynamic Medium of Reference: A new theory of gravitation,"[1] we established that the speed of a photon along a radial trajectory is written

$$c(r) = \frac{dr}{dt} = \varepsilon \left(1 - \frac{2GM}{c_0^2 r}\right) c_0, \quad (7)$$

with the convention $\varepsilon = +1$ if the photon moves away from the center of the black hole and $\varepsilon = -1$ if the photon goes toward the center of the black hole.

Using the expression (6) in Eq. (7), we get

$$dr = \varepsilon\left(1 - \frac{2GM}{c_0^2 r}\right) c_0 \left[dt_{\rm ff} - \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right)^{-1} \frac{V_{\rm flux}}{c_0^2} dr\right],$$

$$dr = \varepsilon\left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right) c_0 dt_{\rm ff} - \varepsilon \frac{V_{\rm flux}}{c_0} dr,$$

$$\left(1 + \varepsilon \frac{V_{\rm flux}}{c_0}\right) dr = \varepsilon \left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right) c_0 dt_{\rm ff}.$$

For a photon moving away from the center of the black hole ($\varepsilon = +1$), we have

$$dr = \left(1 - \frac{V_{\rm flux}}{c_0}\right) c_0 dt_{\rm ff} \text{ that is to say } \frac{dr}{dt_{\rm ff}} = c_0 - V_{\rm flux}. \quad (8)$$

For a photon going toward the center of the black hole ($\varepsilon = -1$), we have

$$dr = -\left(1 + \frac{V_{\rm flux}}{c_0}\right) c_0 dt_{\rm ff} \text{ that is to say } \frac{dr}{dt_{ff}} = -c_0 - V_{\rm flux}. \quad (9)$$

These last two expressions are very important, because they mean that:

- In the "hybrid" coordinate system (dr, $dt_{\rm ff}$), the speed of light is well established simply with regard to its propagation in the medium of centripetal speed $V_{\rm flux} = \sqrt{2GM/r}$.
- Inside the horizon of the black hole ($r < rs$), the speed of the flux of the medium being greater than the speed of light and being centripetal, all photons end their trajectory in the center of the black hole.

In their book on general relativity,[3] the authors establish the formula $dr/dt_{\rm ff} = -\sqrt{2GM/r} \pm 1$ (in reduced units $c = 1$).

For general relativity, the formula $dr/dt_{\rm ff} = -\sqrt{2GM/r} \pm 1$ is only a mathematical equation which shows that the photons in both cases have a negative speed and therefore move toward the center of the black hole.

In the DMR theory, the equation $dr/dt_{\rm ff} = \pm c_0 - V_{\rm flux}$ means much more than that:

- It means that the speed of the photon is to be considered with regard to its medium of propagation which can be in motion (speed of the flux of the medium).
- It shows that, if the speed of the medium is greater than the speed of light, then it prevails over the speed of the photon relative to the medium and the photon finishes its trajectory in the center of the black hole regardless of its initial direction.

V. SPEED OF A PHOTON NEAR THE SCHWARZSCHILD RADIUS

In the **Schwarzschild frame of reference,** we have already seen that

$$c(r) = \frac{dr}{dt} = \pm\left(1 - \frac{2GM}{c_0^2 r}\right) c_0. \quad (10)$$

A photon that leaves from infinity (with regard to the center of the black hole) with a speed c_0 actually arrives near the horizon of the black hole with a speed of zero!

In the **sphere frame of reference**, the speed of the photon is locally

$$\frac{dr_{\rm sphere}}{dt_{\rm sphere}} = \frac{dr}{\sqrt{1 - \frac{2GM}{c_0^2 r}}} \frac{1}{\sqrt{1 - \frac{2GM}{c_0^2 r}} dt} = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \frac{dr}{dt} = \pm c_0. \quad (11)$$

Thus, in the sphere frame of reference, the modulus of the speed of light measured locally by the instruments (material ruler, material clock) is always c_0.

VI. SPEED OF A PARTICLE FALLING TOWARD A BLACK HOLE

In the article *Dynamic Medium of Reference: A new theory of gravitation*,[1] we established that the speed of a material particle along a centripetal radial trajectory is written as

$$V(r) = \frac{dr}{dt} = -\left(1 - \frac{2GM}{c_0^2 r}\right)\sqrt{\frac{2GM}{r}} = -\left(1 - \frac{V_{\rm flux}^2}{c_0^2}\right) V_{\rm flux}. \quad (12)$$

A material particle that starts from infinity (with respect to the black hole) has zero speed, it passes through a maximum speed (in modulus) $V = 2c_0/(3\sqrt{3})$ for $r = 6GM/c_0^2 = 3r_S$ and $V_{\rm flux} = c_0/\sqrt{3}$ and arrives in the vicinity of the horizon of the black hole with zero speed.

In the **sphere frame of reference**, the speed of the material particle is locally

$$\frac{dr_{\rm sphere}}{dt_{\rm sphere}} = \frac{dr}{\sqrt{1 - \frac{2GM}{c_0^2 r}}} \frac{1}{\sqrt{1 - \frac{2GM}{c_0^2 r}} dt} = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \frac{dr}{dt}$$

$$= -\left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \left(1 - \frac{2GM}{c_0^2 r}\right) \sqrt{\frac{2GM}{r}},$$

$$\frac{dr_{\rm sphere}}{dt_{\rm sphere}} = -\sqrt{\frac{2GM}{r}} = -V_{\rm flux}. \quad (13)$$

The speed of a material particle which starts from infinity (relative to the center of the black hole) and arrives near the horizon of the black hole goes from 0 to $-V_{\text{flux}} = -c_0$.

Note:

It is interesting to determine the time it takes for the particle to go from the horizon of the black hole to the center of the black hole.

For this it is necessary to know the energy of the particle.

It is possible to show that the particle has the following energy:

$$E = m_0 c^2 \left(1 - \frac{2GM}{c_0^2 r}\right) \frac{dt}{d\tau}. \tag{14}$$

If we consider a particle starting from infinity with zero speed and therefore of energy $E_\infty = E_0 = m_0 c^2$ then the conservation of energy is written

$$E = m_0 c^2 \left(1 - \frac{2GM}{c_0^2 r}\right) \frac{dt}{d\tau} = E_\infty = E_0 = m_0 c^2 \text{ that is to say}$$

$$\left(1 - \frac{2GM}{c_0^2 r}\right) \frac{dt}{d\tau} = 1 \text{ or furthermore},$$

$$\frac{d\tau}{dt} = 1 - \frac{2GM}{c_0^2 r}. \tag{15}$$

We can then write

$$\frac{dr}{d\tau} = \frac{dr}{dt}\frac{dt}{d\tau} = -\left(1 - \frac{2GM}{c_0^2 r}\right)\sqrt{\frac{2GM}{r}} \cdot \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} = -\sqrt{\frac{2GM}{r}} = -V_{\text{flux}}.$$

The time it takes for the particle to go from the horizon of the black hole to the center of the black hole is given by the following formula:

$$\tau = \int_{r_S}^{0} \frac{d\tau}{dr} dr = \int_{0}^{r_S} \frac{dr}{V_{\text{flux}}} = \int_{0}^{r_S} \sqrt{\frac{r}{2GM}} dr = \left[\frac{2}{3}\frac{r^{3/2}}{\sqrt{2GM}}\right]_0^{r_S} = \frac{2}{3}\frac{r_S^{3/2}}{\sqrt{2GM}} = \frac{2}{3\sqrt{2GM}}\left(\frac{2GM}{c_0^2}\right)^{3/2} = \frac{4GM}{3c_0^3} = \frac{2}{3}\frac{r_S}{c_0}.$$

VII. STUDY OF THE POSSIBILITY OF ORBITS OF A PHOTON AROUND A BLACK HOLE

In the article, *Dynamic Medium of Reference: A new theory of gravitation*,[1] we established that the trajectory of a photon is described by the following equation:

$$\left(K^2 \frac{dr}{dt}\right)^2 + \left(\frac{L}{K \cdot r}\right)^2 = c_0^2, \tag{16}$$

where $K^{-2} = 1 - (2GM/(c_0^2 . r))$ and L is the angular momentum.

Using the relation $dt/dp = (1 - (2GM/(c_0^2 \cdot r)))^{-1}$ (where p is an affine parameter of the geodesic), we obtain the following equation:

$$\left(\frac{dr}{dp}\right)^2 + \left(\frac{L}{K \cdot r}\right)^2 = c_0^2$$

that we can also write

$$\left(\frac{dr}{dp}\right)^2 = c_0^2 - \left(1 - \frac{2GM}{c_0^2 \cdot r}\right)\frac{L^2}{r^2}. \tag{17}$$

From this equation, the books about General Relativity[2,3] deduce the possible cases of trajectories for a photon

We can put down $V^2(r) = \left(1 - \frac{2GM}{c_0^2 \cdot r}\right)\frac{L^2}{r^2} = \frac{L^2}{r^2} - \frac{\alpha}{r^3}$ with $\alpha = \frac{2GML^2}{c_0^2}$.

The speed V is zero at infinity, increases to the value $V_{\max} = Lc_0^2/(3\sqrt{3}.GM) = 2L/(3\sqrt{3}.r_S)$ in $r_{\max} = 3GM/c_0^2 = (3/2)r_S$ and then decreases to 0 in $r_S = 2GM/c_0^2$.

For a photon, there will therefore be only two types of possible orbits:

- $V_{\max} < c_0$: This means that the angular momentum L is "weak." The photon will follow an unstable bound orbit and fall into the black hole,
- $V_{\max} > c_0$: This means that the angular momentum L is "strong." The photon will be able to escape from the gravitational pull of the black hole, after having partially rotated in the equatorial plane.

Although the trajectory of the photons is deflected by the presence of the black hole, there is no such thing as a bound and stable orbit. Thus, it is impossible to consider photons making a lasting movement of revolution around a black hole.

VIII.STUDY OF THE POSSIBLE ORBITS OF A MATERIAL PARTICLE AROUND A BLACK HOLE

In the article, *Dynamic Medium of Reference: A new theory of gravitation*,[1] we established that the trajectory of a material particle is described by the following equation:

$$\left(K^2 \frac{dr}{dt}\right)^2 + \left(\frac{L}{K \cdot r}\right)^2 = \frac{2GM}{r} + C^2 \quad \text{where } K^{-2} = 1 - \frac{2GM}{c_0^2 \cdot r}. \tag{18}$$

Using the relation $d\tau/dt = 1 - (2GM/(c_0^2 \cdot r))$, we get the following equation:

$(dr/d\tau)^2 + (L/(K \cdot r))^2 = (2GM/r) + C^2$ that we can also write

$$\left(\frac{dr}{d\tau}\right)^2 = C^2 + \frac{2GM}{r} - \left(1 - \frac{2GM}{c_0^2 . r}\right)\frac{L^2}{r^2} = C^2 + \frac{2GM}{r} - \frac{L^2}{r^2} + \frac{2GML^2}{c_0^2 . r^3}. \tag{19}$$

The constant C is determined for r tending to infinity. The equation obtained $(dr/d\tau)^2 = C^2$ is then the well-known equation of special relativity $P^2 c_0^2 = E^2 - m_0^2 c_0^4$ with

$P = m(dr/d\tau)$ which we can also write by dividing it by $m^2.{c_0}^2$: $(dr/d\tau)^2 = (E/(mc_0))^2 - (m_0c_0/m)^2 = C^2$.

Equation (19) can then be written

$$\left(mc_0\frac{dr}{d\tau}\right)^2 = E^2 - \left(m_0c_0^2\right)^2 - (mc_0)^2\left[-\frac{2GM}{r} + \frac{L^2}{r^2} - \frac{2GML^2}{c_0^2\cdot r^3}\right]. \quad (20)$$

From this equation, the books about General Relativity[3] deduce the possible cases of trajectories for a material particle.

Note that the smallest bound and stable orbit is a circular orbit of radius $r_{\min} = 6GM/c_0^2 = 3.r_S$.

If a particle reaches a distance $r < 6GM/{c_0}^2$ then its trajectory necessarily becomes an unstable bound orbit and the particle falls into the black hole.

Note: Some authors[2] write Eq. (19) as follows:

$$\left(\frac{dr}{d\tau}\right)^2 + \left(1 - \frac{2GM}{c_0^2.r}\right)\frac{L^2}{r^2} - \frac{2GM}{r} = c_0^2(k^2 - 1) \text{ with } k = \frac{E}{m_0c_0^2}. \quad (21)$$

IX. CONCLUSION

This article has shown that it is possible to find all the results of general relativity concerning black holes without rotation and without load thanks to the theory of DMR (dynamic medium of reference).

In addition, the DMR theory provides insightful new interpretations on the following important points:

- The speed of the flux of the medium $V_{\text{flux}} = \sqrt{2GM/r}$ is greater than the speed of light inside the horizon of a black hole or even much greater than the speed of light at a distance from the center of the black hole much less than the radius of Schwarzschild.
- In the hybrid coordinate system ($dr_{\text{schwarzschild}}$, $dt_{\text{free fall}}$), the speed of light is established in relation to its propagation medium according to the simple formula $dr_{\text{Sch}}/dt_{\text{ff}} = \pm c_0 - V_{\text{flux}}$.
- A photon emitted at an infinite distance from the black hole with the speed c_0 arrives in the vicinity of the horizon of the black hole with a speed really zero in a frame of reference linked to the black hole. And yet the **local measurement** of the speed of the photon carried out with a material clock and a material ruler **remains c_0**, even in the vicinity of the horizon of the black hole.

[1] O. Pignard, Phys. Essays **32**, 422 (2019).
[2] M. Holson, G. Efstathiou, and A. Lasenby, *Relativité Générale* (De Boeck, Paris, France, 2010).
[3] A. Barrau and J. Grain, *Relativité Générale* (Dunod, Paris, France, 2011).