Explanation of the velocity of the stars in the galaxies in the dynamic medium of reference (DMR) theory

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Abstract: The theory of the dynamic medium of reference (DMR) has already been presented in several articles, in particular: "Dynamic medium of reference: A new theory of gravitation" [O. Pignard, Phys. Essays **32**, 422 (2019)]. The article "Theory of the dynamic medium of reference: Exterior case and interior case" [O. Pignard, Phys. Essays **34**, 280 (2021)] gives an explanation and mathematical developments of the gravitational acceleration from atomic nuclei of a massive body. The objective of this article is to explain the velocity of the stars in galaxies within the framework of the DMR theory. The DMR theory proposes to modify the law of gravitation at long distance. The demonstration allowing to obtain the gravitational acceleration makes it possible to establish that: the gravitational acceleration generated by a massive body of mass M one of whose dimensions is much smaller than the other two becomes $\gamma_G = (G/R_L)(M/r)$ for distances greater than a certain limit distance R_L from the massive body, and the gravitational acceleration generated by a massive body of mass M of spherical shape (a star, for example) becomes $\gamma_G = (GM/R_L^2)$ for distances greater than a certain limit distance R_L from the massive body. The first law of gravitation at long-distance $\gamma_G = (G/R_L)(M/r)$ makes it possible to explain a constant star rotation curve from a certain distance from the center of the galaxy. Among 126 galaxies analyzed, this corresponds to the profile of 76 galaxies. For this, it is assumed the existence of dark matter located in the center of the galaxy in the form of a flat disk of thickness much less than its diameter. For rotating stars in this type of galaxy, this causes that beyond the distance R_L from the center of the galaxy, the velocity of the stars becomes constant and equals to $V = \sqrt{GM/R_L}$. The dark matter required by the DMR theory has a mass that is only about 30% that of ordinary matter in a galaxy (stars and interstellar gas) instead of the immense quantities required by current theories. The second law of gravitation at long-distance $\gamma_G = GM/R_L^2$ makes it possible to explain an increasing star rotation curve. Among 126 galaxies analyzed, this corresponds to the profile of 50 galaxies. For this type of galaxy, it is not necessary to assume the existence of dark matter, and all the stars contained in the galaxy are sufficient to explain the star rotation curve. For this type of galaxy, the velocity of the stars increases approximately in proportion to $\sqrt{r}$. Finally, the modifications of the law of gravitation proposed by the DMR theory would also explain the observed values of the deflection of light rays by galaxies (Einstein lenses and rings), which the modified Newtonian dynamics (MOND) theory cannot do.
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Résumé: La théorie du Milieu Dynamique de Référence (DMR) a déjà été présentée dans plusieurs articles, en particulier "Dynamic Medium of Reference: A new theory of gravitation" [Pignard, Phys. Essays **32**, 422 (2019)]. L'article "Theory of the Dynamic Medium of Reference: exterior case and interior case" [Pignard, Phys. Essays **34**, 280 (2021)] donne une explication et les développements mathématiques de l'accélération gravitationnelle à partir des noyaux d'atome d'un corps massif. Le présent article a pour objectif d'expliquer la vitesse des étoiles dans les galaxies dans le cadre de la théorie DMR. La théorie DMR propose de modifier la loi de gravitation à longue distance. La démonstration permettant d'obtenir l'accélération gravitationnelle permet d'établir que : l'accélération gravitationnelle générée par un corps massif de masse M dont l'une des dimensions est bien inférieure aux deux autres devient $\gamma_G = \frac{G}{R_L}\frac{M}{r}$ pour des distances supérieures à une certaine distance limite R_L par rapport au corps massif; l'accélération gravitationnelle générée par un corps massif de masse M de forme sphérique (une étoile par exemple) devient $\gamma_G = \frac{GM}{R_L^2}$ pour des distances supérieures à une certaine distance limite R_L par rapport au corps massif. La première loi de gravitation à longue distance $\gamma_G = \frac{G}{R_L}\frac{M}{r}$ permet d'expliquer une courbe de rotation des étoiles constante à partir d'une certaine distance par rapport au centre de la galaxie. Parmi 126 galaxies analysées, cela correspond au profil de 76 galaxies. Pour cela il est supposé l'existence de matière noire localisée au centre de la galaxie sous forme d'un **disque plat** d'épaisseur bien inférieure à son diamètre. Pour les étoiles en rotation dans ce

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type de galaxie, cela entraine qu'au-delà de la distance R_L par rapport au centre de la galaxie, la vitesse des étoiles devient constante et égale à $V = \sqrt{\frac{GM}{R_L}}$. La matière noire nécessaire dans le cadre de la théorie DMR possède une masse qui n'est que d'environ 30% celle de la matière ordinaire contenue dans la galaxie (étoiles et gaz interstellaire) au lieu des immenses quantités que nécessitent les théories actuelles. La deuxième loi de gravitation à longue distance $\gamma_G = \frac{GM}{R_L^2}$ permet d'expliquer une courbe de rotation des étoiles croissante. Parmi 126 galaxies analysées, cela correspond au profil de 50 galaxies. Pour ce type de galaxie il n'est pas nécessaire de supposer l'existence de matière noire, l'ensemble des étoiles contenues dans la galaxie suffit à expliquer la courbe de rotation des étoiles. Pour les étoiles en rotation dans ce type de galaxie, la vitesse des étoiles croit approximativement de façon proportionnelle à $\sqrt{r}$. Enfin, la modification de la loi de gravitation proposée par la théorie DMR permettrait d'expliquer également les valeurs observées de la déflexion des rayons lumineux par les galaxies (lentilles et anneaux d'Einstein), ce que ne peut pas faire la théorie modified Newtonian dynamics (MOND).

Key words: Dynamic Medium of Reference; Gravitons Field; Gravitation; Velocity of the Stars in the Galaxies; Dark Matter.

I. SUCCINCT PRESENTATION OF THE THEORY OF THE DYNAMIC MEDIUM OF REFERENCE

Important preliminary remark:

This article does not present the theory of the dynamic medium of reference (DMR).

The theory of the Dynamic Medium of Reference has already been presented in several articles,[1–5] in particular "Dynamic medium of reference: A new theory of gravitation"[1] and "Theory of the dynamic medium of reference: Exterior case and interior case,"[4] which is strongly recommended to have read to understand this article.

The theory of the dynamic medium of reference[1] introduces a **dynamic nonmaterial** medium, which is present in the whole Universe.

The characteristics of this medium are:

- This medium enables one to deduce a Preferred Frame of Reference or rather a REFERENCE in the whole Universe and at all scales;
- This REFERENCE enables one to obtain a privileged time. The present moment is universal that is to say the same in the whole Universe;
- This medium is also the medium of propagation of light;
- This medium verifies the principle of reciprocal action:
  - ○ The medium is distorted by matter and energy like the space-time of general relativity;
  - ○ The warping of this medium determines the trajectories of the particles (material particles and light particles).

The presence of a massive body creates a flux of the medium (centripetal that is to say directed toward the center of gravity of the massive body) of speed

$$V_{\text{flux}} = \sqrt{\frac{2GM}{r}} \tag{1}$$

and acceleration

$$\gamma_{\text{flux}} = \frac{GM}{r^2}, \tag{2}$$

where r refers to the distance to the center of gravity of the massive body.

In the framework of the Lorentz/Poincaré theory, in the absence of a gravitational field, material clocks (in the reference frame R) undergo a physical dilatation of their period according to their speed with respect to the Preferred Frame of Reference (PFR) according to the formula

$$T = \gamma.T_0 \quad \text{with} \quad \gamma = \left(1 - \frac{V_{R/\text{PFR}}^2}{c_0^2}\right)^{-1/2}. \tag{3}$$

Within the framework of Lorentz/Poincaré theory, in the absence of a gravitational field, material rulers (in the reference frame R) undergo a physical contraction of their length according to their speed with respect to the Preferred Frame of Reference (PFR) according to the formula

$$L = \frac{L_0}{\gamma} \quad \text{with} \quad \gamma = \left(1 - \frac{V_{R/\text{PFR}}^2}{c_0^2}\right)^{-1/2}. \tag{4}$$

In the presence of a massive body of mass M, the speed of the flux of the medium takes the following expression in the frame of reference linked to the massive body and at a distance r from the center of gravity of the massive body[1]

$$\overrightarrow{V_{\text{flux}}} = -\sqrt{\frac{2GM}{r}}\overrightarrow{u_r}, \tag{5}$$

where $\overrightarrow{u_r}$ denotes the unit radial vector directed toward the exterior of the massive body.

The effects undergone by material clocks and rulers due to their speed with respect to the medium (which allows to define the Preferred Frame of Reference) are the same as the effects they undergo by the centripetal movement of the medium due to a massive body. Since clocks and rulers are assumed to be fixed with respect to the massive body, the centripetal movement of the medium (of speed V_{flux}) with respect to the center of gravity of the massive body can be interpreted as a movement of the clocks and rulers with respect to the medium.

The equivalence between the movement of the clocks and rulers with respect to the medium and the movement of the medium with respect to the clocks and rulers is a new way of stating **the principle of equivalence**.

In the presence of a massive body of mass M, material clocks undergo a physical dilatation of their period according to the following formula:[1]

$$T = T_0.K(r) \quad \text{with} \quad K(r) = \left(1 - \frac{V_{\text{flux}}^2}{c_0^2}\right)^{-1/2} = \left(1 - \frac{2GM}{c_0^2.r}\right)^{-1/2}. \tag{6}$$

In the presence of a massive body of mass M, material rulers undergo a physical contraction of their length according to the following formula:[1]

$$L = \frac{L_0}{K(r)} \quad \text{with} \quad K(r) = \left(1 - \frac{V_{\text{flux}}^2}{c_0^2}\right)^{-1/2} = \left(1 - \frac{2GM}{c_0^2.r}\right)^{-1/2}. \tag{7}$$

Light is slowed down by a gravitational field, and the expression of its speed is[1]

$$c = \frac{c_0}{K\sqrt{1 + (K^2 - 1)\cos^2\beta}}. \tag{8}$$

We name $\beta = \left(\overrightarrow{u_r}, \overrightarrow{c}\right)$ the angle between the unit radial vector $\overrightarrow{u_r}$ and the speed vector of light $\overrightarrow{c}$. The vector $\overrightarrow{c_0}$ represents the speed vector of light if there was not any massive body.

In the case of a radial trajectory of the light, we have the following simple expression:

$$c = \frac{c_0}{n(r)} = \frac{c_0}{K^2(r)}. \tag{9}$$

If we call ε_0 the permittivity and μ_0 the permeability of the medium without gravitational field, then we have $c_0 = (\varepsilon_0\mu_0)^{-1/2}$.

If we call ε the permittivity and μ the permeability of the medium in the presence of a gravitational field created by a massive body of mass M, then we have $c = (\varepsilon\mu)^{-1/2}$ with

$$\varepsilon = \varepsilon_0.\varepsilon_r = \varepsilon_0\left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \tag{10}$$

and

$$\mu = \mu_0.\mu_r = \mu_0\left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}. \tag{11}$$

The refractive index is given by the formula

$$n = \sqrt{\varepsilon_r\mu_r} = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \tag{12}$$

with

$$\varepsilon_r = \varepsilon/\varepsilon_0 = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1} \tag{13}$$

and

$$\mu_r = \mu/\mu_0 = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}. \tag{14}$$

All these formulas show that the medium is related to electricity, magnetism, electromagnetism, and gravitation.

(a) Case of the photon
The exploitation of the previous results makes it possible to establish the Lagrangian formulation of the trajectory of a photon[1]

$$L = K^2\left[c_0^2 - \left(K^2\frac{dr}{dt}\right)^2 - \left(Kr\frac{d\phi}{dt}\right)^2\right] = 0 \tag{15}$$

that we can write

$$\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\phi}{dt}\right)^2 = c_0^2. \tag{16}$$

After the development of calculations, we end up with the following equation:[1]

$$\frac{d^2u}{d\phi^2} + u = \frac{3GM}{c_0^2}u^2 \quad \text{with} \quad u = 1/r. \tag{17}$$

This equation, which is exactly that provided by General Relativity,[6–10] makes it possible to determine the deflection of light rays by a massive body, for example, the Sun, and also by clusters of galaxies (gravitational lens, gravitational mirage, and Einstein ring).

(b) Case of a material particle
The exploitation of the previous results makes it possible to establish the Lagrangian formulation of the trajectory of a material particle[1]

$$c_0^2 - L = K^2\left[\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\phi}{dt}\right)^2 - \frac{2GM}{r} - C^2\right] = 0 \tag{18}$$

that we can write

$$\left(K^2\frac{dr}{dt}\right)^2 + \left(Kr\frac{d\phi}{dt}\right)^2 = \frac{2GM}{r} + C^2 = V_{\text{flux}}^2 + C^2. \tag{19}$$

After the development of calculations, we end up with the following equation:[1]

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{A^2} + \frac{3GM}{c_0^2}u^2 \quad \text{with} \quad u = 1/r. \tag{20}$$

This equation, which is exactly that provided by General Relativity,[6–10] makes it possible to determine the trajectory of the planets of the solar system and, in particular, the precession of the perihelion of Mercury.

II. A POSSIBLE DESCRIPTION OF THE DYNAMIC MEDIUM OF REFERENCE: THE GRAVITONS FIELD

This section gives a possible description of the dynamic medium of reference: the gravitons field.

The gravitons field is based on Le Sage theory BUT it adds many deep changes and evolutions.

Numerous scientists have studied Le Sage theory.[b),11] Just to mention a few of them:

Newton, Huygens, Leibniz, Euler, Laplace, Lord Kelvin, Maxwell, Lorentz, Hilbert, Darwin, Poincaré, and Feynman.

Henri Poincaré has studied this theory and written a synthesis in *Science et Méthode*.[12]

Poincaré sums up the principle of Le Sage theory like this:

> "It is proper to establish a parallel between these considerations and a theory proposed a long time ago in order to explain the universal gravitation. Let's suppose that, in the interplanetary spaces, very tiny particles move in all directions, with very high speeds. A single body in the space will not be affected, apparently, by the impact of these corpuscles, since these impacts are equally divided in all directions. But, if two bodies A and B are in the space, the body B will play the role of a screen and will intercept a part of these corpuscles which would have hit A. Then, the impacts received by A in the opposite direction of the one of B, will not have compensation any longer, or will be imperfectly compensated, and they will push A toward B. Such is Le Sage theory."

It is possible to demonstrate rather easily that the "push" is inversely proportional to the square distance between the two bodies (like the Newton law).

One can also demonstrate that, if the corpuscles are very tiny, the push is approximately proportional to the number of nucleons and so the mass of the body and not the apparent surface of the body.

Moreover, only a tiny fraction of corpuscles hits the atoms of the body, which explains that the push (the gravitational force) is so weak.

The main evolutions of the gravitons field versus Le Sage theory are the following:

- One must not use the notion of impact between the corpuscles and matter.
- One must consider that the corpuscles are **nonmaterial** and constitute a **medium**.
- The total energy of an entity is the sum of its kinetic energy of translation $E_{\text{translation}}$ and of its kinetic energy of rotation about itself E_{rotation}.
- Fundamental law: Conservation of the total energy of an entity: **The total energy of one entity remains constant**

$$E_{\text{total}} = E_{\text{translation}} + E_{\text{rotation}} = \text{constant}. \tag{21}$$

Afterwards, we will call these entities **gravitons** (but these gravitons have nothing to do with the graviton of spin 2 of quantum mechanics).

If the total energy of the gravitons remains constant, then the gravitons do not give energy to the atoms of the Earth and so do not raise the temperature of the Earth contrary to the conclusion made by Poincaré[12] on the original theory of Le Sage.

It is postulated that the gravitons, which interact with the atoms of a massive body, lose some of their kinetic energy of **translation**, which turns into kinetic energy of **rotation**.

The gravitons which interact with the atoms of a massive body lose a part of their translation speed and win some rotation speed and are called **gravitons-spin**.

So a massive body would be a huge "transformer" of "standard gravitons" in "gravitons-spin."

This physical phenomenon does not raise the temperature of a massive body, BUT it has an effect on the **medium**.

The medium undergoes a centripetal flux due to the presence of the massive body.

Indeed, let us consider a reference frame at the surface of the massive body and an elementary volume linked to it.

If one measures the speed vectors of all the gravitons in this elementary volume, the **average of the speed vectors** gives a **resulting speed vector which is centripetal** (because the gravitons-spin coming from the ground have a smallest translation speed than the standard gravitons coming from the sky).

It has been demonstrated in a previous article[4] that the acceleration of the flux of the medium has the following expression $\overrightarrow{\gamma_{\text{flux}}} = -(GM/r^2)\overrightarrow{u_r}$ from which one can deduce that the centripetal speed of the flux of the medium at a distance r from the center of gravity of a massive body of mass M is equal to (measured in a reference frame R linked to the massive body)

$$\overrightarrow{C_{G/R}} = \frac{\sum_{i=1}^{N_G} V_{G/R}}{N_G} = \overrightarrow{V_{\text{flux}}} = -\sqrt{\frac{2GM}{r}}\overrightarrow{u_r}. \tag{22}$$

Definition of the Preferred Frame of Reference based on the entities constituting the medium:

Let us consider a Galilean referential R (a laboratory) and an elementary volume linked to this referential.

In this very small volume, imagine that we can count the entities in it (gravitons) and we can also know the speed vector of each graviton $\overrightarrow{V_{G/R}}$.

Knowing this, it is possible to compute the vectorial average of the speed vectors of the gravitons: $\overrightarrow{C_{G/R}} = (\sum_{i=1}^{N_G} \overrightarrow{V_{G/R}})/N_G$.

This resultant vector means that, at the center of this given elementary volume, the Preferred Frame of Reference moves at the speed $\overrightarrow{C_{G/R}}$ versus the referential R and that the

b) https://en.wikipedia.org/wiki/Le_Sage%27s_theory_of_gravitation referential R (the laboratory) moves at the speed $-\overrightarrow{C_{G/R}}$ versus the Preferred Frame of Reference (defined by the medium), i.e., versus the medium.

Another way to present the Preferred Frame of Reference (PFR) is to define it as the unique referential for which, at any point M in space, we have

$$\overrightarrow{C_{G/\mathrm{PFR}}}(M) = \frac{\sum_{i=1}^{N_G} \overrightarrow{V_{G/\mathrm{PFR}}}}{N_G} = \vec{0}. \quad (23)$$

Figure 1 shows the distortion of the medium due to the Earth.

The flux of the medium:

- Is generated by the presence of the matter of the Earth
- Is always centripetal, i.e., radial, oriented toward the center of gravity of the Earth
- Is maximum at the surface of the Earth and decreases when going away from the Earth
- Has a constant magnitude on every sphere whose center is the one of the Earth
- Gives the impression to follow the Earth in its movement (whatever its speed) because it remains identical to itself.

III. REMINDER ON THE GRAVITATIONAL ACCELERATION IN THE THEORY OF THE DMR

In a previous article,[4] we have determined the acceleration of the flux of the medium created by a massive body at a point M by taking into account the flux of gravitons arriving from all directions in space, and these directions being symbolized by elementary cones of elementary solid angle Ω_e and of section s_e at the distance r from point M. Then we have the relation

$$\Omega_e = \frac{s_e}{r^2}. \quad (24)$$

An elementary cone is associated with only one direction of arrival of the incident gravitons at the given point **M**.

The number of elementary cones describing all directions of 3D space is

$$N_{c(3\mathrm{D})} = \frac{4\pi r^2}{s_e} = \frac{4\pi}{\Omega_e}. \quad (25)$$

The acceleration of the **flux of the medium** is given by the **vector average** of the flux of gravitons

$$\overrightarrow{\gamma_{\mathrm{flux}}} = \frac{1}{N_{\mathrm{tot}}} \sum_{i=1}^{N_{c(3\mathrm{D})}} F_i \overrightarrow{V_{Gi}}. \quad (26)$$

$F_i = N_i/\Delta t$ is the flux of gravitons in a given direction, that is to say, the number of incident gravitons crossing the section of an elementary cone **per second** ($\Delta t = 1$s) and moving toward the apex of the cone.

We also consider that the flux of gravitons is the same in all directions; therefore, in all elementary cones we have

$$F_i = \frac{N_i}{\Delta t} = F_G = \frac{N_G}{\Delta t} = \text{constant}.$$

Regarding N_{tot} we have

$$N_{\mathrm{tot}} = \sum_{i=1}^{N_{c(3D)}} N_i = N_G.N_{c(3D)} = \frac{4\pi N_G}{\Omega_e}. \quad (27)$$

In a previous article,[4] we have demonstrated that the acceleration of the flux of the medium **generated** by a massive body of mass M at a point located at the distance r from the center of gravity of the massive body has the following formula:

$$\overrightarrow{\gamma_{\mathrm{flux}}} = \frac{1}{N_{\mathrm{tot}}} \sum_{i=1}^{N_{c(3\mathrm{D})}} F_i \overrightarrow{V_{Gi}} = \frac{F_G.N_c.N_s.k_s.k_n}{N_{\mathrm{tot}}}.(V_{\mathrm{Gspin}} - V_G)\overrightarrow{u_r}, \quad (28)$$

where

- $N_c = S/s_e = S/\Omega_e.r^2$ is the number of elementary cones intercepted by the surface S of the material body,
- $N_s = e/d$ is the number of elementary slices of thickness the interatomic distance d contained in the thickness e of the material body,
- $k_s = N_n.s_n/d^2$ is the proportion of gravitons for which an atom nucleus is in their path. So k_s is the ratio of the cross section of the nucleons constituting the nucleus of an atom and the interatomic section d^2,

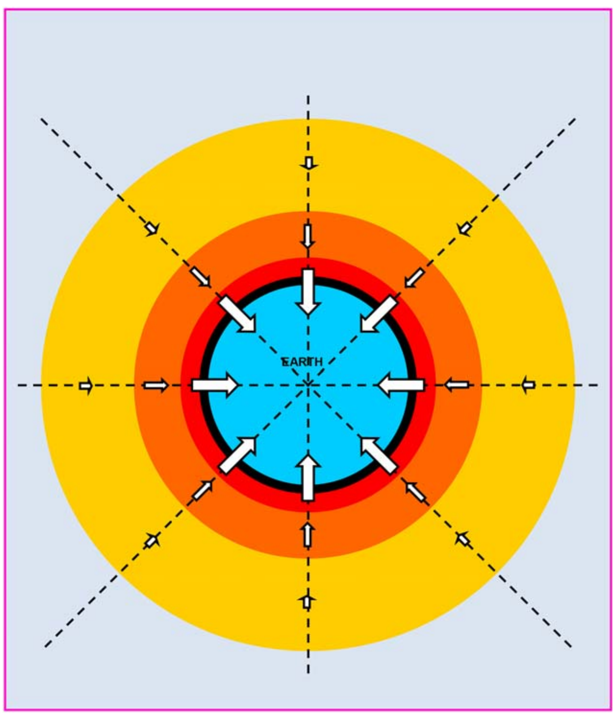

FIG. 1. (Color online) Distortion of the medium due to the Earth.

- m_n is the mass of a nucleon, s_n is the section of a nucleon, and N_n is the number of nucleons in a given atomic nucleus;
- k_n is the proportion of gravitons having encountered an atom nucleus, which have interacted with the nucleus and reemitted in the form of gravitons-spin. 1-k_n is the proportion of gravitons having encountered an atom nucleus and not having interacted with this atom nucleus;
- V_G is the speed of the incident gravitons;
- V_{Gspin} is the speed of the gravitons-spin re-emitted by the nuclei of the atoms of the massive body.

By taking into account the density of the material body $\rho = M/V = M/(S.e) = N_n m_n/d^3$, we have

$$N_c N_s k_s = \frac{S}{\Omega_e.r^2}\frac{e}{d}\frac{N_n.s_n}{d^2} = \frac{S.e}{d^3}\frac{N_n.s_n}{\Omega_e.r^2} = \frac{s_n}{m_n}\frac{M}{\Omega_e.r^2},$$

which makes it possible to obtain

$$\gamma_{\mathrm{flux}} = G\frac{M}{r^2} \quad \text{with} \quad G = \frac{1}{N_{\mathrm{tot}}}\frac{F_G}{\Omega_e}k_n\frac{s_n}{m_n}(V_G - V_{\mathrm{Gspin}}). \tag{29}$$

Finally, using $N_{\mathrm{tot}} = 4\pi N_G/\Omega_e$, we have

$$\overrightarrow{\gamma_{\mathrm{flux}}} = -G\frac{M}{r^2}\overrightarrow{u_r} \quad \text{with} \quad G = \frac{k_n}{4\pi}\frac{s_n}{m_n}(V_G - V_{\mathrm{Gspin}}). \tag{30}$$

In the theory of the Dynamic Medium of Reference, the acceleration of the flux of the medium IS the gravitational acceleration, and we, therefore, find Newton's well-known formula.

IV. VELOCITY OF THE STARS IN A GALAXY

A. Exposure of the problem

The problem concerns the velocity of the stars in a galaxy:

- In a first domain in distance from the center of the galaxy, the velocity of the stars increases with the distance because the mass to be taken into account also increases with the distance [curve (1) in Fig. 2].
- In a second domain in distance, from a certain limit distance R_L, the velocity of the stars should decrease with distance if it followed Newton's laws [curve (2) in Fig. 2].
- However, in this second domain in distance, observations indicate that the velocity of the stars remains approximately constant [curve (3) in Fig. 2].

B. First solution proposed by the dynamic medium of reference theory

The first solution proposed by the DMR theory is based on two essential points:

- The hypothesis of the presence of dark matter in the center of the galaxy in the form of a flat disk of thickness much less than its diameter;

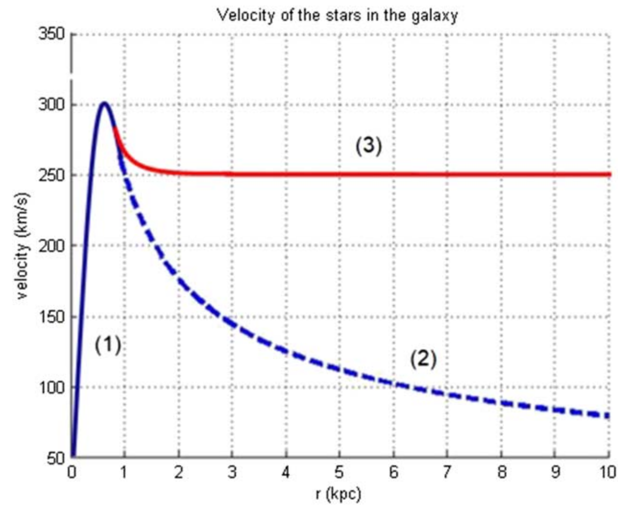

FIG. 2. (Color online) Star rotation curve predicted by Newton's laws (2) and the observed curve (3).

- A law of gravity modified when one is located beyond a certain limit distance R_L from an attracting body of mass M having a dimension much smaller than the other two. We will show that the gravitational acceleration then takes the following form: $\gamma_G = (G/R_L)(M/r)$.

We model dark matter as a flat disk located in the center of the galaxy, of diameter D_d and thickness t_d with $t_d \ll D_d$.

The plane of the disk is the same as the plane of rotation of the galaxy, and therefore, a rotating star sees the disk by its edge.

Figure 3 schematically represents a galaxy and the flat disk of dark matter located at its center in two perpendicular views.

To obtain the gravitational acceleration exerted on a star, it is necessary to determine the number of elementary cones, which have the star in question as their apex and which intercept the slice of the flat disk of dark matter.

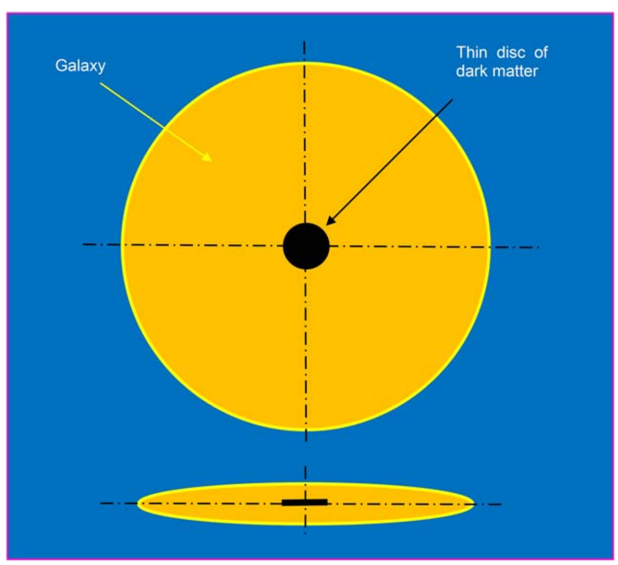

FIG. 3. (Color online) Diagram of a galaxy and the flat disk of dark matter.

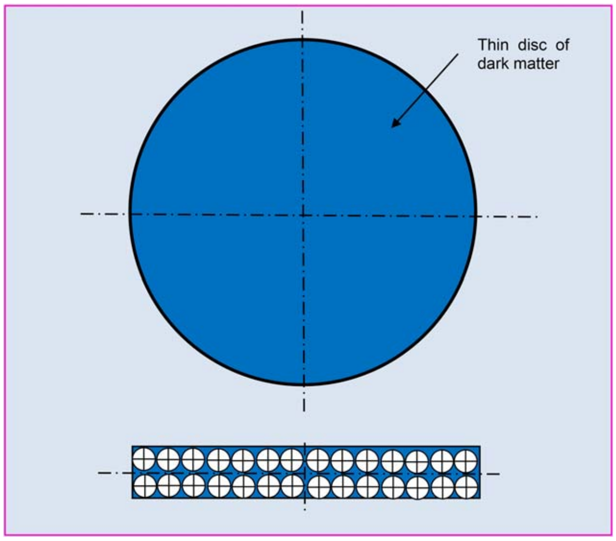

FIG. 4. (Color online) Thin disk of dark matter and elementary cones that intercept its slice.

Figure 4 shows schematically the flat disk of dark matter and the intersection of the elementary cones with the edge of the disk.

We have already seen that the directions of arrival of the flux of gravitons on a point M are symbolized by elementary cones of solid angle $\Omega_e = s_e/r^2$ and of section s_e at the distance r from the point M.

For simplicity, we assume that the cones have a square section with side a_e, and we call ε the opening angle of the cone. We then have the relation

$$\tan\left(\frac{\varepsilon}{2}\right) = \frac{a_e/2}{r} \approx \frac{\varepsilon}{2} \quad \text{which gives us : } a_e = \varepsilon.r \quad \text{and}$$
$$s_e = \Omega_e.r^2 = a_e^2 = \varepsilon^2 r^2.$$

For a star rotating in the main plane of rotation of the galaxy, the flat disk of dark matter is seen by the slice of surface $S_d = D_d.t_d$ and the number of elementary cones intercepting the slice of the disk is

$$N_c = \frac{S_d}{s_e} = \frac{D_d t_d}{\Omega_e.r^2} = \frac{D_d t_d}{\varepsilon^2.r^2} = \frac{D_d}{\varepsilon.r}.\frac{t_d}{\varepsilon.r} = N_{c//}.N_{c\perp}, \quad (31)$$

$$\text{with} \quad N_{c//} = \frac{D_d}{\varepsilon.r} \quad \text{and} \quad N_{c\perp} = \frac{t_d}{\varepsilon.r}. \quad (32)$$

It is considered that the minimum number of elementary cones aligned in a given direction of the material body (for example, the slice of the flat disk at the heart of a galaxy) is **1**. This is due to the fact that the elementary cones mean that the flux of gravitons arriving at a point M cannot come from directions as close as one wants but that the directions of arrival are quantized, the quantum being the elementary cone.

On the other hand, in a given elementary cone, it is considered that the jet of gravitons is very thin, much narrower than the angle ε of the elementary cone. This means that when the observation point M is moved away from a massive body of length L and thickness t, there arrives a distance R_L for which the number of cones in the thickness reaches 1 and the jet of gravitons remains the same in this cone for all distances greater than R_L. The total number of elementary cones describing all the directions of 3D space being determined and equal to $N_{c(3D)} = 4\pi/\Omega_e$, **a single cone** keeps its "weight" compared to the total number of cones $N_{C(3D)}$.

This makes that the law followed by the number of cones $N_{c\perp}$ is (see Fig. 5)

$$\left\{ \begin{array}{ll} N_{c\perp} = \frac{t_d}{\varepsilon.r} & \text{for} \quad r < R_L \\ N_{c\perp} = 1 & \text{for} \quad r \geq R_L \end{array} \right\} \quad (33)$$

where r and $R_L \approx t_d/\varepsilon$ are distances from the center of the galaxy.

Since we assume that $D_d \gg t_d$, the number of cones $N_{c//}$ keeps the formula $N_{c//} = D_d/(\varepsilon.r)$ to the ends of the galaxy.

The total number of cones is, therefore, given by the following formulas (see Fig. 6):

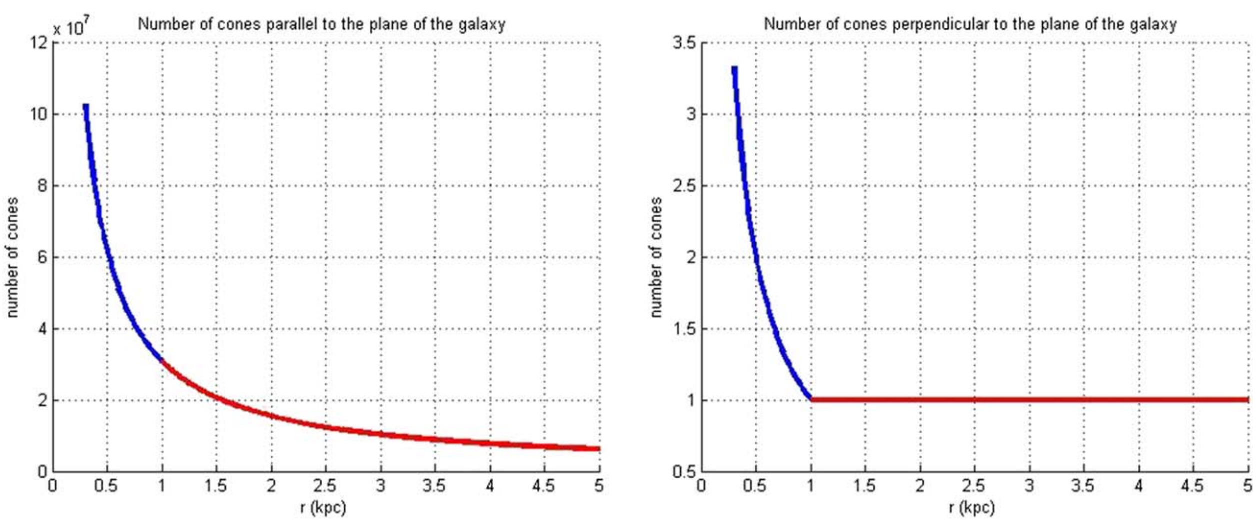

FIG. 5. (Color online) Number of cones parallel and perpendicular to the plane of the galaxy.

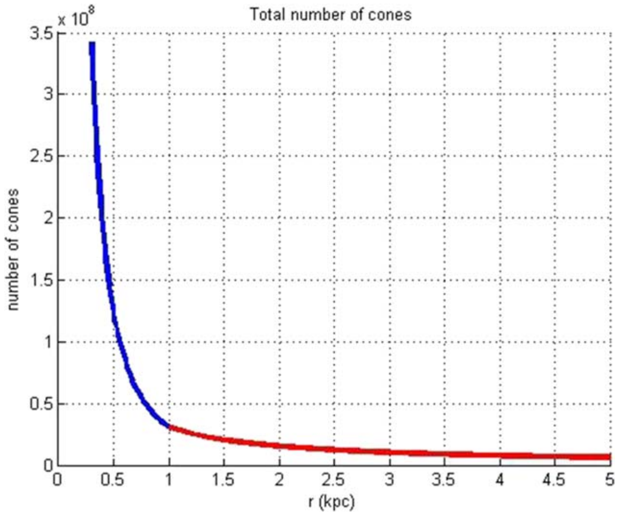

FIG. 6. (Color online) Total number of cones intercepted by the flat disk of dark matter.

$$\left\{\begin{array}{lll} N_c = \dfrac{D_d t_d}{\varepsilon^2 . r^2} & \text{for} & r < R_L \\ N_c = \dfrac{D_d}{\varepsilon . r} & \text{for} & r \geq R_L \end{array}\right\} \tag{34}$$

The acceleration of the flux of the medium (see Fig. 7), which is also the gravitational acceleration, created by the flat disk of dark matter at the center of the galaxy is proportional to the total number of cones N_c.

By using the developments and the relations of Section III, we obtain

$$\left\{\begin{array}{lll} \gamma_{\text{flux}} = G\dfrac{M}{r^2} & \text{for} & r < R_L \\ \gamma_{\text{flux}} = G'\dfrac{M}{r} & \text{for} & r \geq R_L \end{array}\right\} \tag{35}$$

With the limiting condition for $r = R_L$

$$G\frac{M}{R_L^2} = G'\frac{M}{R_L} \quad \text{which} \quad \text{gives} \quad G' = \frac{G}{R_L}. \tag{36}$$

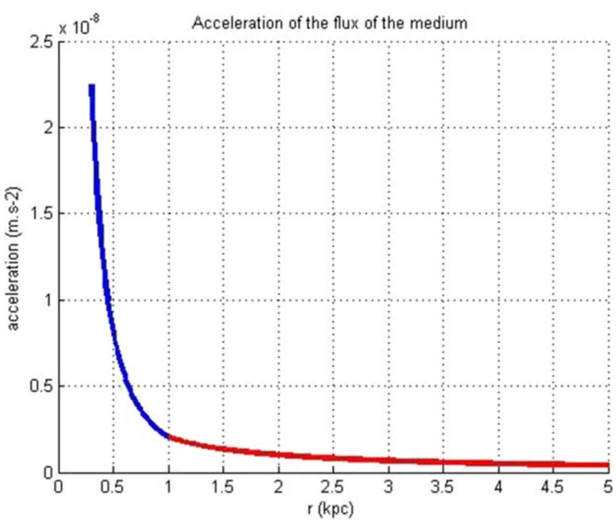

FIG. 7. (Color online) Acceleration of the flux of the medium.

Finally, to determine the velocity of a star, we consider that it describes a circle with a velocity of constant magnitude V and, therefore, of inertial acceleration $\gamma_I = V^2/r$.

In the case $r < R_L$, we have: $\gamma_I = V^2/r$ and $\gamma_G = GM/r^2$, which gives $V = \sqrt{GM/r}$.

In the case $r \geq R_L$, we have: $\gamma_I = V^2/r$ and $\gamma_G = (G/R_L)(M/r)$, which gives $V = \sqrt{GM/R_L}$.

To summarize, we, therefore, have

$$\left\{\begin{array}{lll} V = \sqrt{\dfrac{GM}{r}} & \text{for} & r < R_L \\ V = \sqrt{\dfrac{GM}{R_L}} & \text{for} & r \geq R_L \end{array}\right\} \tag{37}$$

Figure 8 shows three portions of numbered curves:

(1) The velocity of the stars increases as a function of the distance r from the center of the galaxy, because the mass to be taken into account also increases as a function of r.

(2) The velocity of stars decreases as a function of r in accordance with Newton's law of gravitation, but this is not what is observed in reality.

(3) The velocity of the stars remains approximately constant beyond the limit distance R_L, which is well verified by observation.

Important notes:

Only the mass of the flat dark matter disk was taken into account for this study. This is justified by the fact that it is portion (3) that we seek to explain and that the explanation of the observed velocities is based on the presence of the disk of dark matter, the velocity corresponding to ordinary matter being much lower, especially at very great distances beyond 6000 parsecs. Portions (1) and (2) follow Newton's laws and correspond mainly to ordinary matter and dark matter representing only about 30% of ordinary matter (see Section IV C). Precise simulations should be carried out to determine more precisely the gravitational effects of ordinary matter, the gravitational effects of dark matter, and above all to determine exactly the distribution and density of dark matter, while taking into account the gravitational

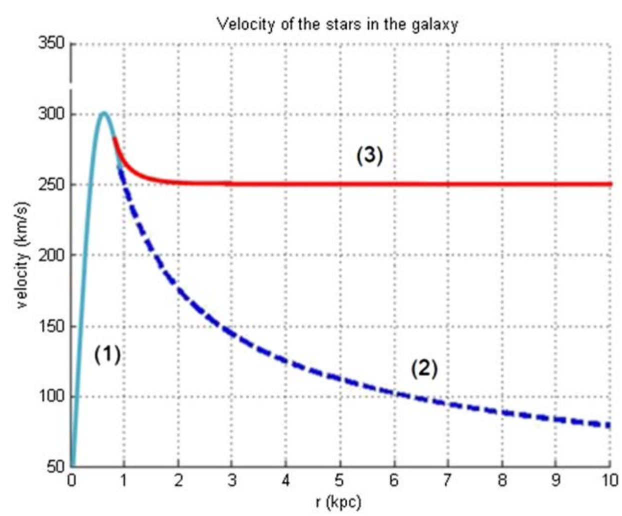

FIG. 8. (Color online) Velocity of the stars in the galaxy.

acceleration $\gamma_G = (G/R_L)(M/r)$ generated by dark matter beyond the limit distance R_L.

If the proposed disk of dark matter is incompatible with the velocity of stars close to the galactic center (for example, less than 1000 AU for the Milky Way), it is possible to propose a distribution in the shape of a crown (or donut) whose inner radius would be much greater than 1000 AU $= 4.848 \times 10^{-3}$ parsec, the outer radius being one parsec (see Section IV C).

If this is not enough to solve the problem, it is possible to draw inspiration from the distribution of dark matter proposed by the standard Lambda-CDM model of cosmology but in much less dense or much less volume.

Indeed, the dark matter predicted by the standard model has abundance at least five times greater than the baryonic matter and represents 85% of the total matter contained in the universe, while the dark matter proposed by this article represents only 30% of ordinary matter or baryonic matter (see Section IV C).

If it is possible to distribute the dark matter of the Standard Model in such a way that it does not disturb stars close to the galactic center, then the dark matter proposed by this article, of mass 19 times lower than that of the Standard Model, must not disturb the stars close to the galactic center either, if it is suitably distributed (while keeping two constraints: one of the dimensions of the volume of dark matter must be much smaller than the other two and the most massive part of this volume must not exceed a certain distance from the galactic center so that the law of gravitation proposed in this part applies).

C. Application case of the first solution on a galaxy close to the milky way

We consider a galaxy of diameter $D_G = 30\,000$ parsecs and thickness $t_G = 300$ parsecs.

We assume that the mass of the galaxy (stars and interstellar gas) is $M_{\text{stars + gas}} = 10^{41}$ kg.

We also suppose that the velocity of the stars deviates from that determined with Newton's law of gravitation for distances to the center of the galaxy greater than $R_L = 1000$ parsecs and that the velocity of the stars then remains approximately constant and equal to $V_L = 250$ km/s.

Regarding the flat disk of dark matter located at the center of the galaxy, its dimensions are obviously not known.

We will take its diameter D_d equal to one parsec and its thickness of the order of the diameter of a star $t_d = 10^9$ m.

We deduce the value of the opening angle of an elementary cone

$$\varepsilon \approx \frac{t_d}{R_L \pm \dfrac{D_d}{2}} \approx \frac{t_d}{R_L} \approx 3.2 \times 10^{-11}\ \text{rad}. \tag{38}$$

We deduce, from the previous data, the mass of the disk of dark matter

$$M_d = V_L^2 . \frac{R_L}{G} = 2.9 \times 10^{40}\ \text{kg} = 14.4 \times 10^9 . M_S, \tag{39}$$

where M_S is the mass of the Sun

and its mean density

$$\rho_d = \frac{M_d}{V_d} = \frac{V_L^2 . R_L / G}{\pi (D_d/2)^2 . t_d} = 0.04\ \text{kg m}^{-3}. \tag{40}$$

Thus, the proposed dark matter, far from having a mass much greater than ordinary matter, only represents the ratio $= M_d/(M_{\text{stars+gaz}}) = (2.9 \times 10^{40})/10^{41} = 29\%$ **of ordinary matter.**

D. Second solution proposed by the dynamic medium of reference theory

The second solution proposed by the DMR theory is based on two essential points:

- This second solution is based only on the visible matter of the galaxy, mainly that of the stars. It does not require the use of dark matter.
- A law of gravity modified when one is located beyond a certain limit distance R_L from an attracting body of mass M, which is a star and takes the following form: $\gamma_G = GM/R_L^2$.

This second solution is developed to explain the star rotation curve of certain galaxies like those of Figures 9 and 10 without having recourse to dark matter.

The essential point is the following:

As a star is an approximately spherical surface body, the number of cones which intersect the apparent surface of the star decreases identically along both axes (what we have called $N_{c//}$ and $N_{c\perp}$) until a single cone is reached.

The total number of cones that intersect a star of apparent surface S is, therefore, given by the following formulas:

$$\left\{ \begin{array}{lll} N_c = \dfrac{S}{\varepsilon^2 . r^2} & \text{for} & r < R_L \\ N_c = 1 & \text{for} & r \geq R_L \end{array} \right\} \tag{41}$$

By using the developments and the relations of Section III, we obtain

$$\left\{ \begin{array}{lll} \gamma_{\text{flux}} = \dfrac{GM}{r^2} & \text{for} & r < R_L \\ \gamma_{\text{flux}} = \dfrac{GM}{R_L^2} & \text{for} & r \geq R_L \end{array} \right\}. \tag{42}$$

These results correspond to the gravitational effect of a single star.

If we call N_{tot} the total number of stars contained in a given galaxy, the acceleration on a given star due to the other $N_{\text{tot}} - 1$ stars is given by the following formula in a Newtonian frame:

$$\overrightarrow{\gamma_{\text{tot}}} = \sum_{i=1}^{N_{\text{tot}}-1} \frac{GM_i}{r_i^2} \overrightarrow{u_i}, \tag{43}$$

where r_i is the distance between the center of the given star and the center of the star of number i, M_i is the mass of the star of number i, and $\overrightarrow{u_i}$ is the unit vector linking the center of the given star and the center of the star of number i.

The projection of this acceleration on the axis (given Star-Center of the galaxy) gives

$$\gamma_{\text{tot}(SC)} = \sum_{i=1}^{N_{\text{tot}}-1} \frac{GM_i}{r_i^2} \cos(\vartheta_i), \tag{44}$$

where ϑ_i is the angle between the vector linking the center of the given star and the center of the galaxy and the vector linking the center of the given star and the center of the star of number i.

We consider that the sum of the accelerations due to the $N_{\text{tot}}-1$ stars projected on the plane perpendicular to (SC) is negligible compared to the projection on the line (SC) because of the supposedly homogeneous distribution of the stars around the line (SC) passing through the center of the galaxy.

We then form two sets with the $N_{\text{tot}}-1$ stars, which act on the considered star:

- The set of N_L stars for which the distance r_i is greater than or equal to the limit distance R_{Li}. We classify these stars from $i=1$ to N_L.
- The set of $N_{\text{tot}} - N_L - 1$ stars for which the distance r_i is less than the limit distance R_{Li}. We classify these stars from $i = N_L + 1$ to $N_{\text{tot}}-1$.

The acceleration $\gamma_{\text{tot(SC)}}$ is, therefore, written

$$\gamma_{\text{tot(SC)}} = \sum_{i=1}^{N_L} \frac{GM_{\text{i}}}{R_{Li}^2} \cos(\vartheta_i) + \sum_{i=N_L+1}^{N_{\text{tot}}-1} \frac{GM_i}{r_i^2} \cos(\vartheta_i). \tag{45}$$

The distance R_{Li} is obtained by the relation: $\tan(\varepsilon/2) \approx \varepsilon/2 \approx (\frac{D_i}{2})/R_{Li}$, where D_i denotes the diameter of the star of number i.

1. Theoretical case where all the stars in the galaxy are identical

If all the stars in the galaxy were identical (same mass M, same diameter D, and therefore same limit distance R_L), we would have

$$\gamma_{\text{tot(SC)}} = \left(\frac{GM}{R_L^2}\right)\left(\sum_{i=1}^{N_L} \cos(\vartheta_i) + \sum_{i=N_L+1}^{N_{\text{tot}}-1} \left(\frac{R_L}{r_i}\right)^2 \cos(\vartheta_i)\right) = \frac{GM}{R_L^2}(A+B), \tag{46}$$

$$\text{with} \quad A = \sum_{i=1}^{N_L} \cos(\vartheta_i), \tag{47}$$

$$\text{and} \quad B = \sum_{i=N_L+1}^{N_{\text{tot}}-1} \left(\frac{R_L}{r_i}\right)^2 \cos(\vartheta_i). \tag{48}$$

Concerning the term B, by definition $r_i < R_L \; \forall \; i$. We, therefore, have $(r_i)_{\max} = R_L$. We can give ourselves a minimum of the order of the parsec, that is to say, $(r_i)_{\min} = 1$ parsec.

We can also take the limit distance of the order of $R_L = 1000$ parsecs.

This means that the term $(R_L/r_i)^2$ is between 1 and 10^6.

However, in a bubble of radius R_L, it can be assumed that the stars are distributed fairly evenly such that their contributions (to the considered star) offset each other and the resultant is quite small.

Concerning the term A, the angle ϑ_i follows a distribution centered on 0 which decreases as it deviates from zero, and this all the more rapidly as the considered star is far from the center of the galaxy.

Finally, the number of stars outside the bubble of radius R_L is much greater than the number of stars inside the bubble.

All these arguments make it legitimate to think that the term A is preponderant over the term B. We would, therefore, have

$$\gamma_{\text{tot(SC)}} \approx A\frac{\text{GM}}{R_L^2}. \tag{49}$$

Finally, using the fact that the radial acceleration of the considered star is also written $\gamma_I = V^2/r$ we get

$$V = \sqrt{\gamma_{\text{tot(SC)}}}\sqrt{r}. \tag{50}$$

Figures 9 and 10 give the velocity of the stars in the galaxies NGC 3109 and NGC 3972:

- with the observed values in red[13] and
- the star rotation curve obtained with the DMR theory in green.

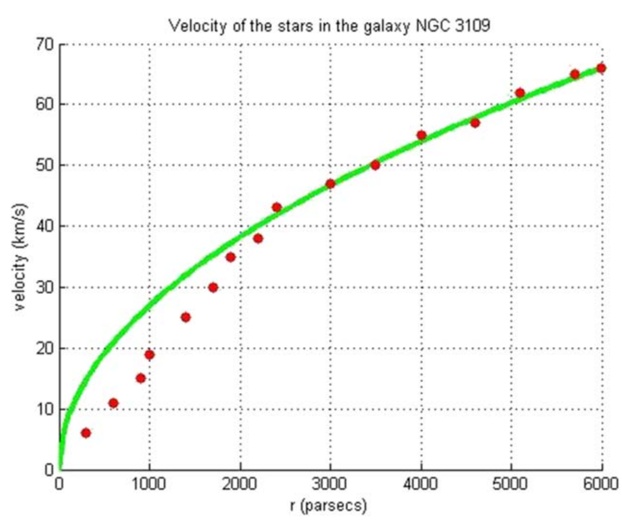

FIG. 9. (Color online) Velocity of the stars in the galaxy NGC 3109 (observed values in RED and curve obtained with the DMR theory in green).

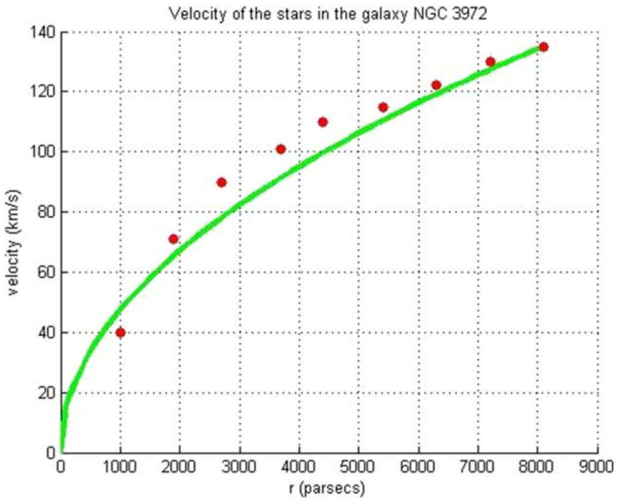

FIG. 10. (Color online) Velocity of the stars in the galaxy NGC 3972 (observed values in RED and curve obtained with the DMR theory in green).

E. Modification of Newton's law of gravitation at long distance

Some authors have long proposed to modify Newton's law of gravity at long distance, for example, Arrigo Finzi, in a 1963 article, "On the validity of Newton's law at a long distance."[14]

Finzi's article was an attempt to solve the longstanding problem of **the stability of clusters of galaxies** by assuming a law of gravitation that implies **a much stronger attraction at a long distance** than that predicted by the law of Newton. The same hypothesis could provide a solution to a number of other problems in different fields of astrophysics.

Indeed, it has been known for a long time[15] that in the great majority of clusters of galaxies **the relative velocities of the member galaxies are very large** and do not seem to be compatible with the stability of the clusters.

It is possible to summarize the different laws of gravitation by the following formula:

$$\gamma = \frac{GM}{R_L^2}\left(\frac{R_L}{r}\right)^{\alpha}, \tag{51}$$

where

- $\alpha = 2$ is Newton's law of gravitation,
- $\alpha = 3/2$ is Finzi's model,
- $\alpha = 1$ is the DMR theory (first model), and
- $\alpha = 0$ is the DMR theory (second model).

"Here R_L is a characteristic length which we shall take to be half a kiloparsec." writes Finzi.

Finzi recognizes the limits of his model:

> "We could not have deduced this behaviour from theoretical considerations. Equation (51) represents merely an attempt to explain some observational facts, and do not claim to have a theoretical foundation."

Conversely, the DMR theory attempts to provide a theoretical justification for the various laws of gravitation proposed in this article.

V. CONCLUSION

There is a tendency among cosmologists to assume that the only viable alternative theories are dark matter or MOND[16] and its relativistic generalization TeVeS (Tensor–Vector–Scalar gravity).[17]

However, mathematician and physicist Ian Stewart writes in one of his books:[18]

> "The distribution of dark matter around galaxies has been plotted by assuming dark matter exists and working out where it has to be to make the rotation curves flat. It generally seems to form two globes of galactic proportions, one above the plane of the galaxy and the other below it, like a giant dumb-bell."
> "The distributions of dark matter don't provide a satisfactory explanation of rotation curves. **Enormous amounts of dark matter** are needed to keep the rotation curve flat out to the large distances observed. The dark matter has to have **unrealistically** large angular momentum, which is inconsistent with the usual theories of galaxy formation. The same rather special initial distribution of dark matter is required in every galaxy, which seems **unlikely**. The **dumb-bell shape is unstable** because it places the additional mass **on the outside** of the galaxy."

Regarding the MOND theory, Ian Stewart writes in the same book:[18]

> "The main problem with MOND, to my mind, is that it puts into its equations what it hopes to get out ; it's like Einstein modifying Newton's law to change the formula near a large mass. Instead, he found a radically new way to think of gravity, the curvature of space-time."

We can add to this that the MOND theory only explains the velocity of stars in galaxies by modifying Newton's second law and adding a minimum acceleration a_0 to it. This works well for stars that have slower and slower acceleration as we move away from the center of the galaxy.

However, this explanation cannot work to explain the observed deviation of light rays by a galaxy (Einstein lenses and rings).

To explain this, we must either add immense amounts of dark matter or modify the law of gravity itself.

The DMR theory proposes to modify the law of gravitation at long distance.

The demonstration allowing to obtain the gravitational acceleration makes it possible to establish that:

- The gravitational acceleration generated by a massive body of mass M one of whose dimensions is much smaller than the other two becomes $\gamma_G = (G/R_L)(M/r)$ for

TABLE I. Acceleration of the flux of the medium according to the type of attracting body and the distance domain and induced velocity of stars in a galaxy.

	Type of body	Distance domain	Acceleration of the flux of the medium	Velocity of stars in a galaxy	Type of galaxy
Newton, DMR theory	Any body of mass M	$r < R_L$	$\gamma = \frac{GM}{r^2}$	$V = \sqrt{\frac{GM}{r}}$	All galaxies
DMR theory	Body with 1 dim $\ll$ 2 other dims	$r > R_L$	$\gamma = \frac{G}{R_L}\frac{M}{r}$	$V = \sqrt{\frac{GM}{R_L}}$	Galaxies of type 2
DMR theory	Spherical body	$r > R_L$	$\gamma = \frac{GM}{R_L^2}$	$V = \sqrt{\gamma_{\text{tot(SC)}} \cdot r}$	Galaxies of type 1

distances greater than a certain limit distance R_L from the massive body.

- The gravitational acceleration generated by a massive body of mass M of spherical shape (a star, for example) becomes $\gamma_G = GM/R_L^2$ for distances greater than a certain limit distance R_L from the massive body.

The first law of gravitation at long-distance $\gamma_G = (G/R_L)(M/r)$ makes it possible to explain a constant star rotation curve from a certain distance from the center of the galaxy, which corresponds to galaxies of type 2 (see Fig. 11).

For this, it is assumed the existence of dark matter located in the center of the galaxy in the form of a flat disk of thickness much less than its diameter.

For rotating stars in this type of galaxy, this causes that beyond the distance R_L from the center of the galaxy, the velocity of the stars becomes constant and equal to $V = \sqrt{GM/R_L}$.

The dark matter required in the context of the DMR theory has a mass which is only about 30% that of the ordinary matter contained in the galaxy (stars and interstellar gas) instead of the immense quantities of dark matter required by the current theories.[18]

Regarding the detection of the proposed dark matter, as it is confined in a flat disk of the order of the parsec in the center of the galaxy, it would not be present in our solar system and therefore could not be detected on Earth.

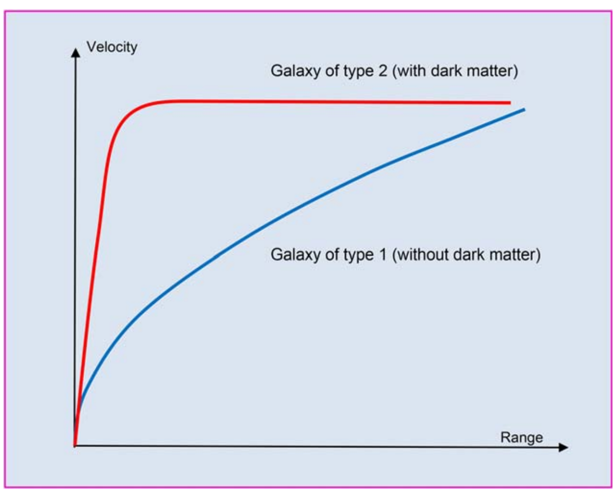

FIG. 11. (Color online) Star rotation curve for galaxies of type 1 and galaxies of type 2.

The second law of gravitation at long-distance $\gamma_G = GM/R_L^2$ explains an increasing star rotation curve corresponding to galaxies of type 1 (see Fig. 11).

For this type of galaxy, it is not necessary to assume the existence of dark matter, and all the stars contained in the galaxy are sufficient to explain the star rotation curve.

For rotating stars in this type of galaxy, the velocity of the stars increases approximately according to the formula $V = \sqrt{\gamma_{\text{tot(SC)}}}\sqrt{r}$, where $\gamma_{\text{tot(SC)}}$ is to be finely evaluated by simulations.

Table I summarizes the different cases obtained with the DMR theory.

The acceleration of the flux of the medium which is none other than the gravitational acceleration is given for several types of massive bodies and two distance domains.

These different accelerations explain the velocity of the stars in galaxies of type 1 and type 2 (see Fig. 11).

Figure 11 shows the star rotation curve in a so-called type 1 galaxy without dark matter and in a so-called type 2 galaxy with dark matter.

It is normal that for two galaxies equivalent from the point of view of ordinary matter (stars, interstellar gas, etc.), the type 2 galaxy (with dark matter) has a star rotation curve above that of the type 1 galaxy (without dark matter) because the excess mass due to dark matter implies a greater velocity of stars.

Finally, the modifications of the law of gravitation justified by the DMR theory could also explain the observed values of deviation of light rays by galaxies or clusters of galaxies (Einstein lenses and rings).

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